

Unemployment Insurance Take-up Rates in an Equilibrium Search Model *

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Abstract

From 1989–2012, on average 23% of those eligible for unemployment insurance (UI) benefits in the US did not collect them. In a search model with matching frictions, the presence of some UI non-collectors, combined with experience rated UI taxes levied on firms implies an inefficiency in non-collector outcomes. This inefficiency is characterized along with the key features of collector vs. non-collector allocations. Specifically, the inefficiency implies that non-collectors transition to employment at a faster rate and a lower wage than the efficient levels. Quantitatively, the inefficiency amounts to 2.82% welfare loss in consumption equivalent terms. This “welfare gain” can be achieved in equilibrium if all firms pay the same UI tax regardless of “experience.” With an endogenous take-up rate, the unemployment rate and average duration of unemployment respond significantly slower to changes in the UI benefit level, relative to the standard model with a 100% take-up rate.

Keywords: unemployment insurance, take-up, matching frictions, search

JEL classification: E61, J32, J64, J65

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1 Introduction

The unemployed *not* collecting benefits they are eligible for may represent the most important issue in the U.S. unemployment insurance system. The existing literature on unemployment insurance (UI) has focused primarily on incentive problems, such as moral hazard, and recently on the effects of UI benefit extension programs (e.g. Nakajima (2012)). In the U.S., from 1989 – 2011, UI fraud and benefit extensions amounted to 2.4% and 13% of total benefit expenditures on average, respectively.¹ According to our analysis, the “unclaimed” benefits from eligible unemployed not collecting benefits are more than the *combined* expenditures on UI fraud and benefit extensions.

In addition, the empirical UI literature has found strong evidence that UI collectors and non-collectors have different unemployment outcomes. Most notably, UI non-collectors tend to have much shorter unemployment durations (see for example Katz and Meyer (1990)). Given that UI collectors and non-collectors have such heterogeneous outcomes, accounting for the take-up decision appears crucial to understanding the impact of UI benefits on equilibrium outcomes. Any change in UI benefits alters not only the behavior and outcomes of UI collectors, but it may also impact how many collect benefits and the behavior and outcomes of the non-collectors. The analysis in this paper shows this to be an important consideration. Our contribution includes a calculation of the fraction of eligible unemployed collecting UI (hereafter “take-up rate”), an equilibrium model incorporating the take-up decision, and an exploration of the implications of unclaimed benefits.

While many U.S. labor market statistics are tabulated and readily available from the Bureau of Labor Statistics (BLS), there exists little information regarding the UI take-up rate. Building on the methodology of Blank and Card (1991), an estimate of the UI take-up rate is calculated

¹UI fraud includes issues such as concealed earnings, insufficient job search, job offer refusals, and quits, among others. See Fuller, Ravikumar, and Zhang (2015) for further details on the issue of UI fraud.

from 1989 – 2012. The calculation uses the March supplement of the Current Population Survey (CPS) along with detailed eligibility criteria by U.S. state. Over the 1989 – 2012 time period, the take-up rate averaged 77%. Given these estimates of the take-up rate, the analysis then develops an equilibrium search model to explain the determinants and implications of this take-up rate.

The take-up decision is captured using a search model with matching frictions where risk-averse workers direct their search to the optimal wage and arrival rate combinations offered by risk-neutral firms.² Workers are heterogenous in their direct utility cost of collecting UI benefits. These costs, along with past UI collections, are private information for the worker. This informational friction combined with the UI financing scheme imply an inefficiency in equilibrium.

Specifically, we model the “experience rating” feature of UI taxes in the U.S., where firms finance UI benefits with the specific tax rate depending on their employees contributions to UI expenditures. We demonstrate analytically how the informational friction and experience rated tax imply that equilibrium is inefficient. Moreover, we show that while UI collectors search efficiently, the inefficiency occurs because non-collector search behavior is distorted.

In the model, firms maximize profits by offering different wage-arrival rate combinations depending on whether or not the worker prefers to collect UI benefits in the event of a future separation. They know the distribution of workers across UI collection costs, but do not observe whether or not the worker has collected in the past. In general, this is not problematic for the firm. They simply offer wage-arrival rate combinations that maximize each type of worker’s expected lifetime utility. This would imply a natural “separating” equilibrium; however, the experience rated tax distorts this natural separation.

The natural separation arises from the effects of different consumption levels with risk-averse

²Rogerson, Shimer, and Wright (2005) offer an overview of a directed search environment and the related literature.

workers. All else equal, UI collectors enjoy higher consumption while unemployed relative to a UI non-collector. Thus, UI collectors search for jobs offering relatively high wages, but longer unemployment durations (*i.e.* slower job arrival rates). In contrast, UI non-collectors prefer to search for relatively low wage jobs with shorter average unemployment durations. With the UI tax accumulating only to those firms hiring a future UI collector, for an equivalent arrival rate, the UI non-collector has a higher wage than a collector. Thus, for some range of UI collection costs a UI collector may find it beneficial to collect benefits but search for the non-collector wage (and associated arrival rate). Indeed, we show that this is true for any positive level of experience rating.

We characterize how firms manage this incentive problem. In equilibrium, it implies that UI non-collector wages and arrival rates are distorted from the efficient levels. Relative to efficiency, non-collectors have a lower wage and shorter unemployment duration. This distortion makes the non-collector job (wage and arrival rate) less appealing to a UI collector.

To understand the magnitude of the inefficiency, we use our estimates of the UI take-up rate to calibrate the model to U.S. data and quantify the welfare consequences of the distortions. Given the observed take-up rate and level of experience rating, the non-collector distortion amounts to a welfare loss of 2.82% in consumption equivalent terms. The welfare loss is measured relative to an economy with UI taxes but no experience rating. Taxes are levied equally on firms regardless of their “experience” with insured unemployment. Our analytical results show that such an economy is efficient. As the level of experience rating increases, so does the welfare cost of the inefficiency. Moving to a fully experience rated system increases the welfare loss to 4.12%.

The analysis also focuses on how incorporating the UI take-up decision affects the impact of UI benefits on equilibrium outcomes. That is, how does an increase in UI benefits affect moments such as the unemployment rate and average duration of unemployment. We find that allowing for an

endogenous take-up rate has important implications. Specifically, while an increase in UI benefits does imply an increase in both the unemployment rate and average duration of unemployment, these two moments respond slower relative to a standard search model with a fixed 100% take-up rate. This occurs in part because the average duration of unemployment actually decreases for non-collectors when UI benefits increase.

Indeed, the effect of UI benefits on the search behavior of UI collectors and non-collectors represents an important aspect of our analysis. As discussed above, UI collectors prefer longer durations and higher wages relative to non-collectors. [Acemoglu and Shimer \(1999, 2000\)](#) present a related finding: as UI benefits increase, workers prefer to search for higher wage jobs arriving less frequently.³ They show how this feature may lead to UI benefits increasing productivity. We abstract from the productivity dimension, focusing instead on modeling the take-up decision and its implications. On this dimension, we explore an inefficiency in allocations not present in the models of [Acemoglu and Shimer \(1999, 2000\)](#).

This paper also contributes to the literature examining the implications of experience rated UI taxes. Our model focuses primarily on the implications of experience rating for hiring and UI take-up. As the level of experience rating increases both the unemployment rate and average duration of unemployment decrease. Moreover, increasing experience rating has unambiguously negative effects, as it increases the inefficiency in non-collector allocations. Our results regarding experience rating are generally in contrast to the existing literature, which finds positive welfare implications of a fully experience rated system.

Specifically, the existing literature analyzing the effects of experience rating focuses on the potential separation effects. For example, by making firms responsible for at least the majority of a

³[Marimon and Zilibotti \(1999\)](#) also examine the implications of UI benefits on productivity, focusing on how UI benefit differences may explain differences in U.S. and European labor market outcomes.

worker's UI expenditures, a firm may prefer to reduce hours worked as opposed to a full separation. Feldstein (1976) and Topel (1983), for example, examine this dimension. Albrecht and Vroman (1999) and Cahuc and Malherbet (2004) represent two examples where the effects of experience rating are examined in an equilibrium search model with matching frictions. Wang and Williamson (2002) also examine the effects of experience rating in a general equilibrium setting as well as examining an optimal UI scheme with experience rating. Their analysis allows experience rating to potentially impact both job retention and job finding rates. All of the aforementioned studies find positive effects of a fully experienced rated system and find that moving to such a system reduces the unemployment rate.

The remainder of the paper proceeds as follows. In Section 2 we present data on experience rating in the U.S. system and our procedure for estimating the take-up rate. Section 3 describes the model, and Section 4 we analytically derive the key properties of equilibrium. Section 5 presents the calibration and quantitative welfare results. Section 5.5 explores the effects of UI benefits and Section 6 concludes.

2 Evidence on take-up rates

This section has two objectives: a description of the experience rating feature of the U.S. system and a description of our take-up rate estimation and exploration of its key features.

2.1 Unemployment Insurance System in the U.S.

Unemployment benefits in the U.S. are financed by a tax levied on firms. This tax levied is “experienced rated”. Firms pay a tax rate that is positively correlated with their contribution to *insured* unemployment in their particular U.S. state. For example, a firm that has never separated from a worker who collects benefits pays a lower tax rate than a firm that has frequent layoffs who collect benefits. Note, for the firm’s tax rate, it does not matter how frequently they separate from workers, but how frequently they separate from workers who collect benefits.

In addition, each state has a maximum and minimum tax rate. This implies that the U.S. system has “partial” experience rating. A firm at the maximum tax rate will not see its tax rate increase in response to an increase in its contribution to UI expenditures. In this regard, firms at the maximum tax rate are subsidized. Similarly, a firm at the minimum tax rate may contribute to the UI fund while rarely sending an employee to insured unemployment. Indeed, this partial experience rating feature may have important impacts on labor market outcomes. For example, the work of [Feldstein \(1976\)](#) and [Topel \(1983\)](#) explore how this feature may influence firm layoff decisions. While we abstract here from the firm layoff decision, this previous work suggests important potential impacts from partial experience rating on this dimension.

2.2 Quantifying the Extent of Experience Rating

As described above, the UI tax scheme potentially implies a marginal cost to firms of separating from a worker. If the worker separates and enters insured unemployment, the firm’s tax rate increases based on that worker’s contribution to UI expenditures in the state. Note, since firms are only partially experience rated (some firms pay a maximum or minimum rate), the average marginal cost to a firm (across the economy) is less than one. Given the variance across states in the precise calculation of firm tax rates, determining a metric to compare the degree of experience rating across states remains difficult. To aid in this comparison, the U.S. Department of Labor publishes data related to the financing of the UI system. From 1989 – 2004, the Department of Labor tabulated an index referred to as the “Experience Rating Index,” or ERI.

The ERI is a measure of how “responsible” employers in a given state are for the benefits charged by their former employees.⁴ In other words, the ERI is one approximation of the extent of the “firing cost” associated with experience rated UI taxes. Our model below adopts this interpretation.⁵

Figure 1 displays the average value of the ERI across all U.S. states from 1989 – 2004, and from 2008 – 2012. From 2005 – 2007, the ERI is not tabulated by the U.S. Department of Labor. The tabulations from 2008 – 2012 use a different calculation. Since 2008, the preferred metric is now “The Average Increase in an Employer’s Per employee Tax for Incurring Benefit Charges Equivalent to 1% of its Taxable Payroll.” The idea of this index is to calculate the average additional cost an employer will incur if it sends an employee to insured unemployment.⁶ Over the 1989 – 2004

⁴Appendix B provides more details on the specific calculations and differences across states

⁵Topel (1983) uses the specific tax formula for several states and calculates an estimate of the marginal cost to a firm of sending a worker to insured employment. In general, Topel (1983) estimates a slightly higher marginal cost than we calibrate to using the ERI.

⁶From the Significant Measures of State UI Tax Systems, published by the BLS (Bureau of Labor Statistics), it is calculated as: “The difference between the maximum per employee cost at the tax base and the minimum per employee cost, divided by the difference between the experience rating percent (either Reserve Ratio or Benefit Ratio) corresponding to the maximum statutory tax rate and the experience rating percent corresponding to the minimum statutory tax rate.”

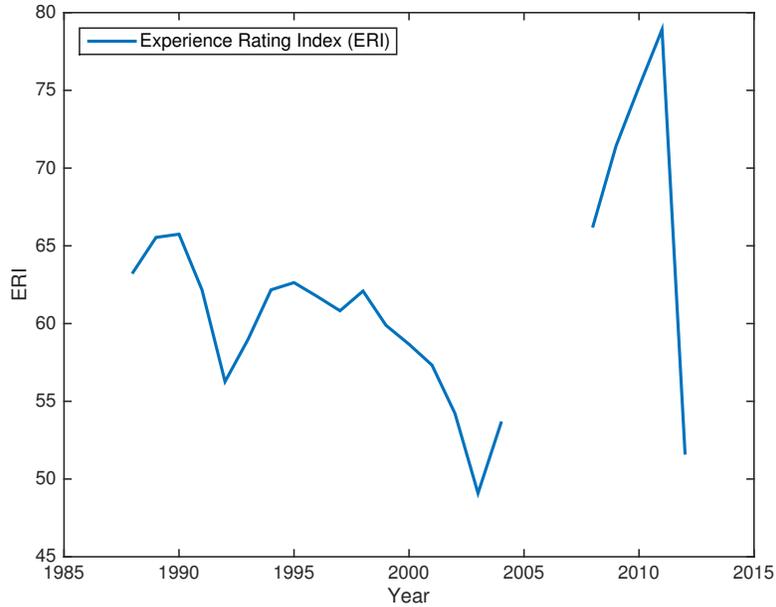


Figure 1: Experience Rating Index Over Time

Notes: The figure displays the average value of the ERI over the period from 1988 – 2004, and from 2008 – 2012. The vertical axis is in percent.

period, the average ERI is 60.02.

2.3 Take-up rate estimates

While many statistics and data on the labor market are readily available for public use, there exists little information on take-up rates of unemployment insurance. There is data on the characteristics of the insured unemployed (those collecting benefits), as well as data on the ratio of insured unemployed to total unemployed (hereafter IUR). While this provides some characterization of the take-up rate, the IUR does not control for eligibility. That is, many of the unemployed are not eligible to collect benefits. To calculate the *take-up rate*, we first find the fraction of unemployed agents who are currently eligible to collect, and then take the ratio to insured unemployed to *eligible* unemployed.

We follow a method similar to [Blank and Card \(1991\)](#). Specifically, we start with IUR data,

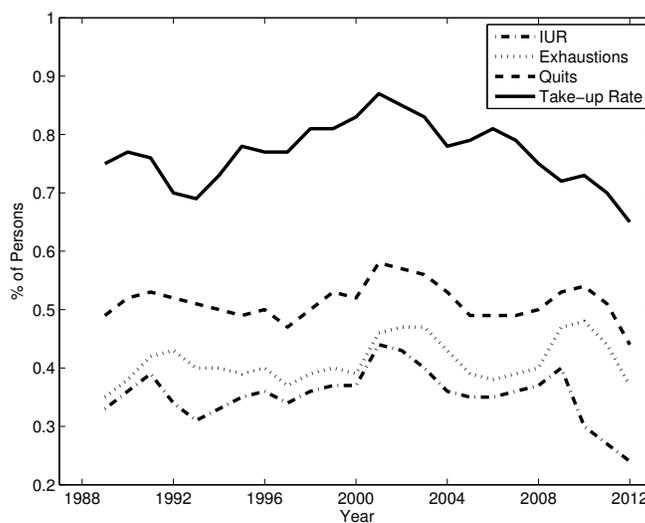


Figure 2: Take-up Rates by Eligibility Criteria Over Time

The bottom line labeled “IUR” is the ratio of insured unemployed to total unemployed. As the lines progress, unemployed individuals are eliminated from the denominator based on different eligibility criteria. “Exhaustions” removes those ineligible because they exhausted their benefits and “Quits” removes those who are ineligible because they quit the job. The jump from the “Quits” line to the “Take-up Rate” line occurs when those unemployed who are ineligible because they do not meet the monetary requirements are removed. Thus, the “Take-up Rate” line plots the fraction of eligible unemployed collecting benefits.

which refers to those collecting “Regular Program” benefits. These are the 26 weeks of benefits primarily financed by each state, and exclude any extended benefit programs financed by the state or federal government. We use the IUR series tabulated by the U.S. Department of Labor, which can be found at: <http://workforcesecurity.doleta.gov/unemploy/chartbook.asp>.

To determine the fraction of unemployed eligible for regular program benefits, we use data from the March Supplement of the CPS, along with the specific eligibility criteria of each state, for each year from 1989 – 2012.⁷ The take-up rate is calculated as the ratio of the IUR to the Fraction of Unemployed Eligible. Figure 2 displays the IUR, our estimate of the take-up rate, and a decomposition of the different eligibility criteria.

Eligibility depends primarily on three factors, all of which are determined at the state level.

⁷Blank and Card (1991) and Anderson and Meyer (1997) provide estimates of the take-up rate prior to 1989.

First, as mentioned above, there is a fixed duration that an individual may collect benefits for. In the majority of states, regular program benefits have a potential duration of 26 weeks. In all of the years studied, Massachusetts and Washington have a maximum potential benefit duration of 30 weeks. Beginning in 2004, Montana has a maximum potential benefit duration of 28 weeks. Again, these potential durations refer to the *Regular Program* benefits, and thus exclude any extended benefit programs. Of course, being unemployed for longer than 26 weeks does not necessarily make an individual ineligible. The key issue is whether or not the individual exhausted their regular program benefits (*i.e.* already collected benefits for the maximum potential duration).

To control for this eligibility criteria, we use the information in the March CPS about whether an individual collected benefits in the previous year or not. If an individual is unemployed in March of a given year and has expired regular program benefits, then they have been unemployed for longer than 26 weeks (accounting for differences in Massachusetts, Washington, and Montana where applicable) and must have collected benefits in the previous year.⁸ We consider such individuals as ineligible. In addition to the maximum length of benefits, many states also have a minimum waiting period, typically 1 week, and we control for this criteria where applicable.

In Figure 2 we plot the take-up rate along with a decomposition of the three eligibility criteria. The line labeled “TUR” is the ratio of insured unemployed to total unemployed. As the lines progress, we remove some unemployed individuals from the denominator (total unemployed) until we reach the number of unemployed eligible for benefits.

The line labeled “Exhaustions” removes from the total unemployed those who have exhausted their benefits. On average, over the period from 1989 – 2012, 11% of those ineligible for benefits

⁸In some states, certain individuals may have potential benefit durations less than 26 weeks, depending on their particular circumstances. The most common potential duration, however, is 26 weeks.

were deemed to have exhausted benefits.⁹ As expected, this criteria has a cyclical contribution, with more individuals exhausting benefits during periods of high unemployment. For example, in 2010, 31% of those ineligible were due to exhausted benefits.

The nature of the separation leading to the spell of unemployment represents the second element of eligibility criteria. Specifically, in most states, individuals who quit their previous job, or were fired for cause, are not eligible to collect benefits. In certain years, Georgia is an exception and does allow job leavers (quits) to collect benefits, but they face an increased waiting period before eligible. This criteria is intended to limit benefits to only those individuals who have lost their job through no fault of their own. In the CPS data, we can eliminate quits; however, we cannot determine whether or not the agent was fired for cause. We also use information on an individual's industry to focus only on covered employment. As in [Blank and Card \(1991\)](#), we eliminate postal workers, federal public administration workers, and ex-service persons, as this group is not eligible to collect UI benefits. In [Figure 2](#), the line labeled "Quits" shows the contribution of this eligibility criteria. On average, 19% were ineligible because they quit their previous job.

Finally, there exist monetary eligibility requirements. These require an agent to have accumulated a sufficient amount of earnings in a specified "base-period," or worked a minimum number of weeks.¹⁰ To estimate monetary eligibility, we use the earnings information contained in the March CPS, along with the state-level monetary eligibility requirements. Such monetary requirements vary significantly across states.

There are several different varieties of monetary eligibility. Perhaps the most standard is to require base period wages that exceed some multiple of the weekly benefit amount. The weekly

⁹Note, this fraction will not match with the BLS series calculating the fraction of collectors who exhaust benefits, but is strongly correlated with it. What we measure are those individuals who exhaust benefits *and* are still unemployed as of March in that year. This fraction is necessarily below the fraction that exhaust benefits.

¹⁰The base-period differs across states. Many use a year, while others use two quarters. The base-period is used both to determine monetary eligibility and to calculate the specific benefit an individual is entitled to.

benefit amount (WBA) is the benefit the worker would be entitled to, which is based on these previous earnings. For example, in 1989, Colorado required base period wages to exceed 40 times the WBA. Determining eligibility with this type of criteria also requires estimating the WBA for the individual; each state also has specific rules determining the WBA given earnings. As another example, in 1989, California required base period earnings of at least \$1,200.

High Quarter Earnings (HQE) represents an important quantity for monetary eligibility in some states. This also represents a drawback to using the March CPS earnings information (Blank and Card (1991) also discuss these drawbacks). Since it only details earnings during the previous year, HQE cannot be determined. In some states, eligibility is based on earnings outside of the HQE. For example, in 1989, Georgia required base period earnings greater than 1.5 times the HQE. In such cases, we are unable to determine monetary eligibility.¹¹

In Figure 2, moving from the “Quits” line to the take-up rate displays the contribution of monetary requirements to the number of unemployed deemed ineligible for benefits. On average, 71% of those deemed ineligible failed to satisfy their state’s monetary eligibility requirements.

Figure 3 plots the IUR and the fraction of unemployed eligible for benefits. When the two lines move closer together, the take-up rate increases, and it decreases when the lines diverge. The Fraction of Unemployed Eligible for benefits displays a similar cyclical pattern to the IUR.

2.4 Reasons for Non-collection

The estimates above imply that from 1989 – 2012, on average 23% of those eligible for UI benefits did not collect them. One may ask what are the reasons for non-collection? Given the

¹¹Using weeks worked represents one possible way to proxy for this type of eligibility. For example, in the case of Georgia above, we could require total weeks worked in the previous year to exceed 19.5 ($1.5 * 13$). This assumes constant earnings over the year, and simply requires the individual to have worked more than one quarter. We have implemented this alternative and it has a negligible impact on the fraction of unemployed eligible for benefits.

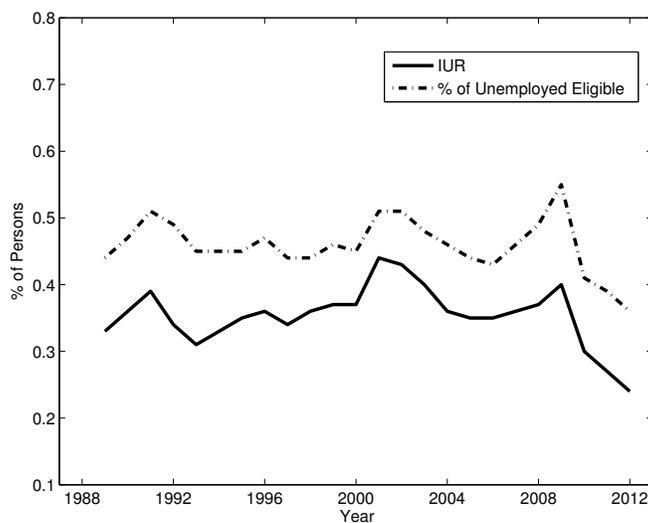


Figure 3: Insured Unemployed and Fraction of Unemployed Eligible

eligibility criteria discussed above, there clearly exist some costs to applying for UI benefits and thus verifying eligibility. The exact nature of these costs and the exact reason(s) for non-collection has not been determined or well-documented in the literature. [Anderson and Meyer \(1997\)](#) cite some survey results offering possible reasons for non-collection, but no particular reason dominates. Of course, an individual does not collect UI benefits if they believe the net benefit to doing so is negative. In our model below, we simply model this as a per-period utility cost of collecting UI benefits.¹²

3 Model

The economy consists of a unit-measure of infinitely-lived, risk-averse workers, and a large measure of risk-neutral firms. Time is continuous and goes on forever, and both agents and firms discount the future at rate $r > 0$. Workers have preferences over consumption, with flow utility

¹²[Auray and Fuller \(2016\)](#) explore possible micro-foundations for these utility costs associated with participating in the UI system.

given by $h(c)$, where c represents consumption. Firms are composed of a single job, either filled or vacant. Vacant firms are free to enter and pay a flow cost, $\gamma > 0$, to advertise a vacancy. Vacant firms produce no output. The flow output of a firm with a filled job is given by y . There are several components of the model to specify. We begin by describing the key features of how UI is modeled.

3.1 Unemployment Insurance

As discussed in Section 2.4, we assume that applying for UI benefits and verifying eligibility imply a flow utility cost to workers who collect UI. Furthermore, we assume that this flow utility cost is additively separable and occurs each period the worker collects UI benefits. Workers are heterogeneous with respect to their costs of collecting UI benefits, which is denoted by ε . Let $F(\varepsilon)$ denote the distribution of workers over ε . If a worker collects UI benefits, they receive flow consumption b and we assume they last forever, or until the worker transitions to employment. If the worker decides not to collect UI benefits while unemployed, they receive flow consumption d , where $b > d$. Thus, each period of unemployment a UI-collector with collection cost ε receives flow utility of $h(b) - \varepsilon$ while a non-collector receives $h(d)$.

Unemployment benefits are financed by lump-sum taxes levied on firms. These taxes are experienced in the following manner. If a firm separates from a worker who collects UI benefits, the firm pays a flow cost of τ . The value of τ determines the marginal cost to a firm of sending a worker to insured unemployment.

In addition, we assume that the worker's UI collection cost, ε , is private information (known only to the worker). Moreover, the firm does not observe whether or not the worker collected UI in the past. Given ε is permanent, knowledge of UI collection history would enable the firm to infer ε . The firm does know the distribution of ε , $F(\varepsilon)$.

3.2 Wages and Matching

We assume directed search. Firms post combinations of wages and queue lengths, and workers direct their search to the combination that maximizes their expected lifetime utility (*i.e.* wages are determined by competitive search, see [Moen \(1997\)](#) or [Acemoglu and Shimer \(1999\)](#) for a similar formulation of the environment).

There exists a matching function, denoted $m(u, v)$, describing the number of matches formed between the v vacancies and u unemployed workers. We assume standard properties, *i.e.* m is continuous, strictly increasing, strictly concave (with respect to each of its arguments), and exhibits constant returns to scale. Furthermore, $m(0, \cdot) = m(\cdot, 0) = 0$ and $m(\infty, \cdot) = m(\cdot, \infty) = \infty$. Let $q = \frac{u}{v}$ denote the “queue” length.

Given this matching technology, a vacancy is filled with Poisson arrival rate $m(\frac{u}{v}, 1)$. Similarly, an unemployed worker finds a job according to a Poisson process with arrival rate $m(1, \frac{v}{u})$. Let $\alpha_E(q) = m(\frac{u}{v}, 1)$ and $\alpha_W(q) = m(1, \frac{v}{u})$ denote the vacancy filling and job finding rates, respectively. Filled jobs receive negative idiosyncratic productivity shocks rendering the match unprofitable with a Poisson arrival rate λ .

Finally, since workers are heterogenous with respect to their costs of collecting UI, firms will prefer to offer different combinations of wages and arrival rates. As some workers decide to collect UI benefits (and thus incur the cost of collecting) and some do not, we denote the different allocations offered by firms as follows. Let $q_U(\varepsilon)$ denote the queue length for a worker who collects UI benefits with collection cost ε , and q_N the queue length for an unemployed non-collector. Similarly, let $w_U(\varepsilon)$ and w_N denote the wages for unemployed collectors and non-collectors, respectively. Below we characterize these quantities in equilibrium.

3.3 Value Functions

Unemployed workers can be in two possible states depending on whether or not they collect unemployment benefits. Denote unemployed collecting UI by $i = U$ and not-collecting by $i = N$. The worker decides which unemployment state to enter the instant a separation occurs, when the worker transitions from employment to unemployment.

3.3.1 Workers

Let $U(\varepsilon)$ denote the expected value of searching for a job with wage and queue length combination $(w_U(\varepsilon), q_U(\varepsilon))$ for an unemployed worker collecting UI with cost of collecting ε . Similarly, let N denote the lifetime utility for the worker if not collecting UI, and $W(\varepsilon)$ the lifetime utility of employment. Given this, the value functions are given by:

$$rU(\varepsilon) = h(b) - \varepsilon + \alpha_w(q_U(\varepsilon)) [W_U(\varepsilon) - U(\varepsilon)] \quad (1)$$

$$rN = h(d) + \alpha_w(q_N) [W_N - N] \quad (2)$$

$$rW_\tau(\varepsilon) = h(w(\varepsilon)) + \lambda (\max\{U(\varepsilon), N\} - W_\tau(\varepsilon)) \quad (3)$$

where $\tau \in \{N, U\}$. Equation (1) implies that an unemployed worker collecting benefits receives instantaneous flow utility $h(b)$ from unemployment compensation, and with arrival rate $\alpha_w(q_U(\varepsilon))$ the worker matches with a firm and transitions to employment. Equation (2) has a similar interpretation for an unemployed worker not collecting. Finally, equation (3) states that an employed worker receives instantaneous flow utility $h(w(\varepsilon))$ and with Poisson arrival rate λ , the job dissolves. If the job dissolves, the worker decides whether or not to collect unemployment benefits. Notice, since the costs of collecting are permanent, in the steady state, if a worker prefers to collect

UI benefits once, he always prefers to. Thus, there are two employed value functions, each with a distinct wage function, $w_\tau(\varepsilon), \tau \in \{N, U\}$ (although for $\tau = N$, the wage and value functions do not depend on ε):

$$rW_U(\varepsilon) = h[w_U(\varepsilon)] + \lambda(U(\varepsilon) - W_U(\varepsilon)) \quad (4)$$

$$rW_N = h[w_N] + \lambda(N - W_N) \quad (5)$$

It is useful to have closed form solutions for $U(\varepsilon)$ and N in the analysis below. Towards this end, using Equations (1) and (4) and Equations (2) and (5) to solve for $U(\varepsilon)$ and N , respectively, gives,

$$U(\varepsilon) = \left(\frac{1}{r + \lambda + \alpha_W(q_U(\varepsilon))} \right) \left((r + \lambda)(h(b) - \varepsilon) + \alpha_W(q_U(\varepsilon))h(w_U(\varepsilon)) \right) \quad (6)$$

$$N = \left(\frac{1}{r + \lambda + \alpha_W(q_N)} \right) \left([(r + \lambda)h(d) + \alpha_W(q_N)h(w_N)] \right) \quad (7)$$

To ensure that worker's indifference curves in (q, w) space are strictly convex requires the following assumption:

Assumption 1 *The matching function satisfies:*

$$\left[\alpha'_W(q) \right]^2 + \alpha_W(q) \left[2[\alpha'_W(q)]^2 - \alpha''_W(q)\alpha_W(q) \right] > 0 \quad (8)$$

This assumption represents a sufficient condition for the worker's indifference curve (in (q, w) space) to be strictly convex. It is generally true for a range of q (for large enough q), and was satisfied in the relevant range for all of the numerical examples computed in Section 5. Another way to view

this assumption is as a sufficient condition for the value functions $U(q)$ and $N(q)$ to be strictly concave in q .

3.3.2 Firms

Denote by $V_i(\varepsilon)$ the value of a vacancy, and by $J_i(\varepsilon)$ the value of a matched firm for $i \in \{N, U\}$. Notice, these value functions depend on the worker's utility cost of UI collection, ε . This utility cost influences whether or not the worker collects UI benefits if separated from the firm. Given the experience rating of UI finance in the model, this decision by the worker affects the value of a filled job for the firm. Specifically, the value function for a vacancy is given by:

$$rV_i(\varepsilon) = -\kappa + \alpha_E[q_i(\varepsilon)] [J_i(\varepsilon) - V_i(\varepsilon)] \quad (9)$$

Similarly, the Bellman equation describing the value of a matched firm is given by:

$$rJ_i(\varepsilon) = y - w_i(\varepsilon) + \lambda [-\chi_i\tau + (V_i(\varepsilon) - J_i(\varepsilon))] \quad (10)$$

where χ_i is an indicator function with $\chi_U = 1$ and $\chi_N = 0$. That is, a firm employing a worker who prefers to collect UI benefits pays a flow tax τ if a separation occurs. Given free entry, $V = 0$, we have,

$$J_i(\varepsilon) = \frac{y - w_i(\varepsilon)}{r + \lambda} - \lambda\chi_i\tau \quad (11)$$

Plugging Equation (11) into Equation (9) under free entry and solving for $w_i(\varepsilon)$ yields,

$$w_i(\varepsilon) = y - \frac{\kappa(r + \lambda)}{\alpha_E(q_i(\varepsilon))} - \lambda\chi_i\tau \quad (12)$$

Thus, if a worker collects UI benefits ($\chi_U = 1$), they receive a lower wage for an equivalent queue length relative to a UI non-collector, which compensates the firm for the resulting increase in taxes. Below we characterize how this experience rating feature combined with private information distorts equilibrium allocations.

3.3.3 Determination of Wages and Queue Lengths

In a standard directed search environment, firms post wages and workers direct their search towards the combination of wages and queue lengths that maximize their utility. Now, since free entry implies that firms earn zero profits in equilibrium, the wage is determined by Equation (12), and depends on whether or not the worker collects UI benefits. Thus, the problem of the firm becomes to optimally choose the queue length, $q_i(\varepsilon)$. For now, assume that there exists a unique ε^* such that $U(\varepsilon^*) = N$, and $U(\varepsilon) > N, \forall \varepsilon < \varepsilon^*$; that is, there exists a unique cut-off value for the decision to collect UI benefits for not. Below we show this is indeed true in equilibrium.

For UI collectors, queue lengths and wages are determined as the solution to the following problem:

$$rU^*(\varepsilon) = \max_{q_U(\varepsilon)} \left(h(b) - \varepsilon \right) \left[\frac{r + \lambda}{r + \lambda + \alpha_W(q_U(\varepsilon))} \right] + \frac{\alpha_W(q_U(\varepsilon))}{r + \lambda + \alpha_W(q_U(\varepsilon))} h(w_U(\varepsilon)) \quad (13)$$

$$\text{s.t. } w_U(\varepsilon) = y - \frac{\gamma(r + \lambda)}{\alpha_E(q_U(\varepsilon))} - \lambda\tau \quad (14)$$

According to this problem, the firm simply chooses $q_U(\varepsilon)$ to maximize the workers utility. This

ensures that a worker of type ε prefers to search for the $w_U(\varepsilon)$ job instead of $w_U(\tilde{\varepsilon})$, for any $\tilde{\varepsilon} \neq \varepsilon$.

Since not everyone collects UI benefits, however, the problem in Equation (13) applies only for $\varepsilon \leq \varepsilon^*$. Thus, we need to also establish the wage and queue length combinations for UI non-collectors. Non-collectors do not pay the utility cost of collecting, so their utility is independent of ε ; as a result, the queue length and wage offered to non-collectors is also independent of ε . This implies that there exists only one wage and queue length offered to non-collectors. Typically, this combination is determined analogously to $q_U(\varepsilon)$ and $w_U(\varepsilon)$: the firm maximizes the workers utility subject to the zero profit condition. Specifically,

$$rN^* = \max_{q_N} h(d) \left[\frac{r + \lambda}{r + \lambda + \alpha_W(q_N)} \right] + \frac{\alpha_W(q_N)}{r + \lambda + \alpha_W(q_N)} h(w_N) \quad (15)$$

$$\text{s.t. } w_N = y - \frac{\gamma(r + \lambda)}{\alpha_E(q_N)} \quad (16)$$

For notational purposes, let q_N^* denote the arg max to Equation (15).

There exists an potential issue with this determination of wages and queue lengths. Specifically, since ε and previous UI collection status is unobservable by a firm, there is no way to prevent a current UI collector from searching for the non-collector wage. Why may a UI collector prefer this option? This occurs because of the experience rated tax. The tax, τ , implies that for a given queue length q , $w_U(\varepsilon) < w_N$ (follows from Equation (12)). For example, this introduces the possibility that a UI collector could search for a higher wage job that arrives faster than the $w_U(\varepsilon)$, strictly dominating it. Define $\tilde{U}(\varepsilon)$ as the value function for a UI collector who deviates and searches for the non-collector job. This is given by,

$$\tilde{U}(\varepsilon) = \frac{1}{r + \lambda + \alpha_W(q_N)} [(r + \lambda)(h(b) - \varepsilon) + \alpha_W(q_N)h(w_N)] \quad (17)$$

In order for the function $q_U(\varepsilon)$ and q_N to be viable in equilibrium, they must satisfy the constraint that:

$$U(\varepsilon) \geq \tilde{U}(\varepsilon) \quad (18)$$

If this constraint is violated, a UI collector prefers to search for the non-collector job, and q_N and firm profits are no longer consistent (since the firm opening a “non-collector” job will pay taxes when those workers separate and collect UI benefits). To maintain an equilibrium allocation, it turns out the non-collector allocation must be altered to satisfy the constraint. The following Remark addresses this issue.

Remark 1 *Equilibrium requires the non-collector job (q_N, w_N) to be altered to satisfy the constraint in Equation (18). Potentially, one may consider altering the UI collector jobs, $(q_U(\varepsilon), w_U(\varepsilon))$ to ensure the constraint is satisfied; however, this remains infeasible. This is true because the firm posting the $(q_U(\varepsilon), w_U(\varepsilon))$ has no control over a worker’s utility when they deviate. That is, this firm does not control q_N and therefore can only alter the utility a worker receives from applying to their job. Since $q_U(\varepsilon)$ already maximizes a type ε worker’s utility, no other possible $q_U(\varepsilon)$ can increase $U(\varepsilon)$ to satisfy the constraint. As a result, q_N must be altered from q_N^* .*

Given the constraint in Equation (18), define \tilde{q}_N as the equilibrium q_N , which is determined as:

$$\tilde{q}_N = \arg \max_{q_N} \frac{1}{r + \lambda + \alpha_W(q_N)} [(r + \lambda)h(d) + \alpha_W(q_N)h(w_N)] \quad (19)$$

$$\text{s.t. } w_N = y - \frac{\gamma(r + \lambda)}{q_N \alpha_W(q_N)} \quad (20)$$

$$U(\varepsilon) \geq \tilde{U}(\varepsilon), \forall \varepsilon \leq \varepsilon^* \quad (21)$$

If the constraint in Equation (21) does not bind in equilibrium, then $\tilde{q}_N = q_N^*$ and equilibrium is efficient. Below we show that this is true when $\tau = 0$, but if $\tau > 0$ $\tilde{q}_N \neq q_N^*$. Towards this end, the following result is useful to define and characterize the equilibrium below.

Proposition 1 *The crossing point, ε^* such that $U(\varepsilon) \geq N, \forall \varepsilon \leq \varepsilon^*$, is unique.*

That is, given the value functions described above, there exists a unique critical value of ε , ε^* , such that for $\varepsilon \leq \varepsilon^*$ the worker prefers to collect UI. For $\varepsilon > \varepsilon^*$ the worker does not collect UI.

3.3.4 Labor market flows and stocks

Our description of equilibrium also requires the flow equations associated with the measures of workers in the different employment and unemployment states. Denote the number of unemployed workers collecting UI for each ε by $n_U^u(\varepsilon)$, and the number of unemployed not collecting UI by n_N^u . Similarly, let $n_U^E(\varepsilon)$ denote the number of employed workers in state $\tau = U$ (*i.e.* will collect UI if separated) and n_N^E the number of employed workers in state $\tau = N$ (*i.e.* will not collect UI if separated).

To obtain a steady state equilibrium, for each ε the flows of workers into and out of employment must be equal. Since the market segments along ε , with $\varepsilon \leq \varepsilon^*$ collecting UI benefits and all others not, we can characterize these equilibrium flow equations as:

$$\lambda n_U^E(\varepsilon) = \alpha_W [q_U(\varepsilon)] n_U^u(\varepsilon) \quad (22)$$

$$f(\varepsilon) = n_U^E(\varepsilon) + n_U^u(\varepsilon) \quad (23)$$

Equation (22) states that for UI collectors, the flow of workers in and out of employment is equal and equation (23) ensures that the total measure of workers across the two employment states adds

up to the population fraction, or $f(\varepsilon)$. Similarly, for $\varepsilon > \varepsilon^*$:

$$\lambda n_N^E = \alpha_W(q_N)n_N^u \quad (24)$$

$$1 - F(\varepsilon^*) = n_N^E + n_N^u \quad (25)$$

Given these flow equations, further denote the total number of employed (unemployed) with $\varepsilon \leq \varepsilon^*$ by $N_U^j = \int_0^{\varepsilon^*} n_U^j(\varepsilon)d\varepsilon, j = E, u$. Further let $N_N^j \equiv n_N^j, j = E, u$. Then, for $\varepsilon \leq \varepsilon^*$, equations (22) and (23) give

$$N_U^u = \int_0^{\varepsilon^*} \frac{f(\varepsilon)\lambda}{\lambda + \alpha_W[q_U(\varepsilon)]} d\varepsilon \quad (26)$$

Similarly we have:

$$N_N^u = \frac{[1 - F(\varepsilon)]\lambda}{\lambda + \alpha_W(q_N)} \quad (27)$$

Thus, the unemployment rate for this economy is given by $u = N_U^u + N_N^u$. The take-up rate is the fraction of eligible unemployed who collect UI benefits. Since we assume all workers remain eligible for UI benefits, this is given by:

$$\text{TUR} = \frac{N_U^u}{u} \quad (28)$$

3.3.5 Definition of Equilibrium

In this section we define equilibrium for the economy described above. Unemployed workers direct their search to the posted wage and queue length combinations that maximize their expected lifetime utility. Firms that hire UI collectors determine the queue length that maximizes profits,

or equivalently that maximizes worker utility, subject to a zero-profit condition. Firms hiring non-collectors face an additional constraint: their queue length must be such that a UI collector does not search for that wage. Formally:

Definition 1 *An equilibrium is defined as a set of wages, queue lengths, value functions, and a cut-off value ε^* , $\{w_U(\varepsilon), w_N, q_U^*(\varepsilon), \tilde{q}_N, U, N, \varepsilon^*\}$ such that:*

1. *Profit maximization:*

(a) $w_U(\varepsilon)$ solves Equation (14), and

(b) w_N solves Equation (20).

2. *Optimal job application:*

(a) $q_U^*(\varepsilon)$ is the solution to Equation (13), and

(b) \tilde{q}_N solves Equation (19) subject to Equation (21)

3. *Consistency:*

$$U(\varepsilon) \geq N, \forall \varepsilon \leq \varepsilon^*,$$

$$U(\varepsilon) < N, \forall \varepsilon > \varepsilon^*.$$

4 Properties of Equilibrium

This section characterizes the key properties of equilibrium in the economy described above. To begin, we characterize the economy with no experience rating, or $\tau = 0$. In this economy, there exists no incentive problem as both non-collectors and collectors are offered the same wage for an equivalent queue length. As a result, equilibrium is efficient.

Proposition 2 *If $\tau = 0$, there exists a unique equilibrium that is efficient, i.e. $\tilde{q}_N = q_N^*$. In equilibrium the following is true:*

(i). $\varepsilon^* = h(b) - h(d)$.

(ii). $q_U^*(\varepsilon) \geq q_N^*, \forall \varepsilon \leq \varepsilon^*$, with equality at $\varepsilon = \varepsilon^*$.

(iii). $w_U^*(\varepsilon) \geq w_N^*, \forall \varepsilon \leq \varepsilon^*$, with equality at $\varepsilon = \varepsilon^*$.

When there exists no experience rating, the incentive problem imposed by Equation (21) is no longer relevant. This is true because the wage functions $w_U(\varepsilon)$ and w_N coincide for any given q ; as a result, any queue length and wage combination offered to UI collectors is feasible to be offered to non-collectors with no potential for deviations. Thus, q_N and $q_U(\varepsilon)$ are determined according to Equations (13) and (15). The equilibrium cut-off value for collecting UI benefits, ε^* occurs at the point where the flow utility from collecting UI benefits, $h(b) - \varepsilon$, is just equal to the flow utility of a non-collector, $h(d)$.

According to property (ii), UI collectors have longer unemployment durations and higher wages, on average. In general, as UI collectors have higher consumption relative to non-collectors, they prefer higher wages and longer unemployment durations. This is similar to the standard result that higher UI benefits imply longer unemployment durations (see Acemoglu and Shimer (1999) for example). Moreover, as ε increases, the utility difference between employment and unemployment increases. This implies the worker prefers to trade-off lower wages for a faster job arrival rate as the net benefit provided by UI is reduced.

It is also worth noting that Proposition 2 is also true for $\tau > 0$, *assuming* that all firms are taxed equally. That is, a firm is taxed regardless of whether or not a separated worker collects UI benefits. With equal taxation, again the wage functions $w_U(\varepsilon)$ and w_N coincide for any given

q and the equilibrium will be efficient. In Section 5 we explore the welfare implications of this no-experience rating equilibrium.

We now analyze the equilibrium properties with experience rating, or $\tau > 0$ for only firms separating from UI collectors. In this case, we show that the constraint in Equation (21) must bind, making the equilibrium inefficient. The following Proposition summarizes this result:

Proposition 3 *For $\tau > 0$, the following is true:*

- (i). $\tilde{q}_N \neq q_N^*$ and the equilibrium is inefficient.
- (ii). \tilde{q}_N is determined by $U(\varepsilon^*) = N(\tilde{q}_N)$, where $N(q)$ is given by Equation (7).
- (iii). $\varepsilon^* = h(b) - h(d)$.

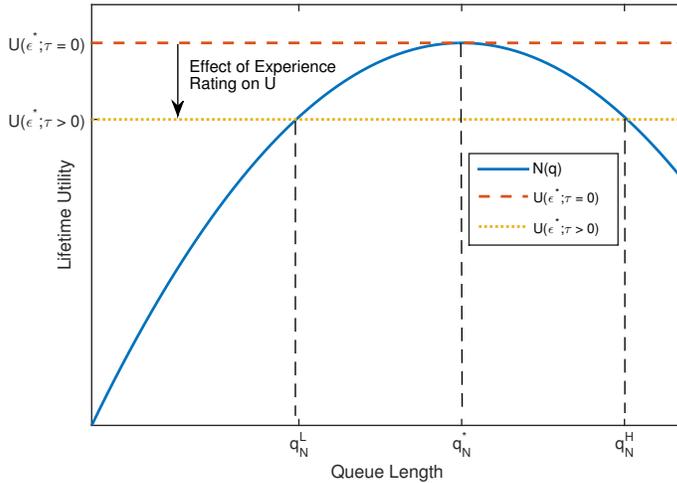


Figure 4: Effect of Experience Rating on Equilibrium and Efficiency

Examining the difference between the $\tau = 0$ and $\tau > 0$ cases helps illuminate the key frictions imposed by experience rating and the intuition for Proposition 3. When $\tau = 0$ the equilibrium queue length for non-collectors is $\tilde{q}_N = q_N^*$ so that $N = N^*$ defined in Equation (15). At $\varepsilon = \varepsilon^* = h(b) - h(d)$, $U(\varepsilon^*; \tau = 0) = N^* = \tilde{U}(\varepsilon^*)$. This is shown in Figure 4 where $U(\varepsilon^*; \tau = 0)$ is tangent

to $N(q)$ at $q = q_N^*$. It is important to notice that $\tilde{U}(\varepsilon^*) = N$ always holds given any q_N , including the equilibrium q_N , and that $\tilde{U}(\varepsilon^*)$ is independent from τ ; as a result, $\tilde{U}(\varepsilon^*) = N$ holds for both $\tau = 0$ and $\tau > 0$.¹³

Now, when $\tau > 0$ the utility of collectors decreases for each ε , since the wage w_U decreases (see Equation (14)). That is, $U(\varepsilon; \tau = 0) > U(\varepsilon; \tau > 0)$. Thus, $U(\varepsilon^*; \tau > 0) < U(\varepsilon^*; \tau = 0) = N^* = \tilde{U}(\varepsilon^*)$. Figure 4 shows how increasing τ from zero decreases $U(\varepsilon^*; \tau)$, and how this now relates to $N(q)$. Recall, $N(q)$ does not depend on τ (see Equation (7)). At ε^* if q_N^* is the equilibrium queue length for non-collectors, Equation (18) is violated at $\varepsilon = \varepsilon^*$, as a collector can achieve $\tilde{U}(\varepsilon^*) = U(\varepsilon^*; \tau = 0)$ at q_N^* . Figure 5(a) shows the relationship between $U(\varepsilon)$ and $\tilde{U}(\varepsilon; q_N^*)$. Indeed, under (w_N^*, q_N^*) the constraint is violated for all $\varepsilon \leq \varepsilon^*$.

Given this, the equilibrium queue length can no longer remain at the efficient level. Specifically, it must change to decrease N until the constraint is satisfied at ε^* . In Figure 4, we show this change from $\tau = 0$ to $\tau > 0$, and Figure 5(b) displays the resulting relationship between $U(\varepsilon)$ and $\tilde{U}(\varepsilon; \tilde{q}_N)$. Notice, \tilde{q}_N is altered just until the constraint is satisfied at $\varepsilon = \varepsilon^*$; this ensures the constraint is satisfied for all UI collectors, and that among \tilde{q}_N satisfying Equation (18), utility is maximized for non-collectors.

From Figure 4, it is clear that a strictly concave $N(q)$ implies that when $\tau > 0$, there exists two potential equilibrium values of \tilde{q}_N . This is true because $N(q)$ has a unique maximum, so ensuring that $N(\tilde{q}_N) = U(\varepsilon^*; \tau > 0)$ implies two possible equilibrium non-collector queue lengths. The following Corollary formalizes this result.

Corollary 1 *For $\tau > 0$, there are two possible equilibrium values of \tilde{q}_N satisfying $\tilde{q}_N^L < q_N^* < \tilde{q}_N^H$.*

¹³This is true as at $\varepsilon = \varepsilon^*$, $h(b) - \varepsilon^* = h(d)$. Thus, a collector and non-collector have the same flow utility. If they search for the same wage and arrival rate combination, they thus have the same expected lifetime utility; *i.e.* $\tilde{U}(\varepsilon^*) = N$. Since N does not depend on τ (all else equal), this is true for any τ .

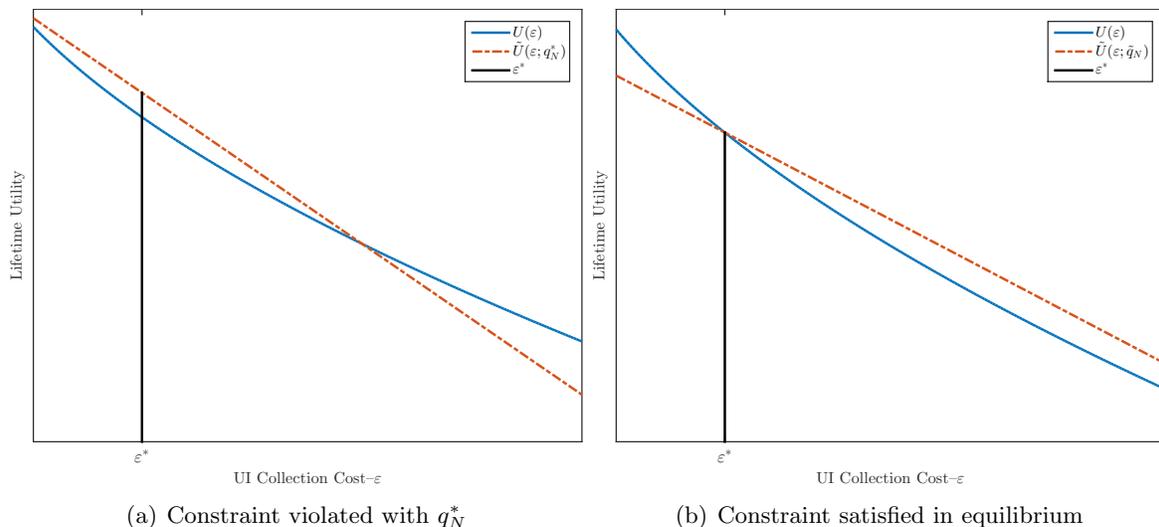


Figure 5: Incentive Constraint in Equilibrium

The left figure plots $U(\varepsilon)$ (solid curve) and $\tilde{U}(\varepsilon; q_N^*)$ (dashed curve), the utility of a collector searching for (w_N^*, q_N^*) . The constraint is violated in this case, as for $\varepsilon \leq \varepsilon^*$, $\tilde{U}(\varepsilon; q_N^*) > U(\varepsilon)$. The right figure shows how the constraint is satisfied for $\varepsilon \leq \varepsilon^*$ under the equilibrium \tilde{q}_N . This \tilde{q}_N is determined as the value satisfying $U(\varepsilon^*) = N(q_N)$. The vertical line in both graphs corresponds to $\varepsilon = \varepsilon^*$.

Moreover, $\tilde{q}_N^L < q_U(\varepsilon)$ and $w_N(\tilde{q}_N^L) < w_U(\varepsilon)$ for all $\varepsilon \leq \varepsilon^*$ implying that non-collectors have a shorter unemployment duration and lower wage than UI collectors.

While indeed there exist two possible \tilde{q}_N 's, our empirical analysis below rules out \tilde{q}_N^H . Specifically, under \tilde{q}_N^H , the job-arrival rate for UI collectors exceeds that of non-collectors. This is contrary to the empirical evidence on the effects of UI benefits, all of which suggest a UI collector has a longer average duration of unemployment relative to a non-collector (for example see [Katz and Meyer \(1990\)](#)).

Having characterized the key properties of the equilibrium with endogenous take-up rates, we now turn towards quantifying these implications.

5 Empirical analysis

In this section, we present a quantitative analysis of the aforementioned model and equilibrium.

Our calibration focuses on the time period from 1989 – 2012.

5.1 Calibration

The model described in Section 3 leaves the following parameters to be determined: β , r , b , d , λ , γ , $F(\varepsilon)$, τ , and functional forms for the matching function, m , and the utility function, h .

The time period is set to one month, so a per-annum risk-free interest rate of 4% implies $r = 0.0033$. The utility function is given by

$$h(c) = \frac{c^{1-\phi} - 1}{1 - \phi} \quad (29)$$

For the coefficient of relative risk aversion, ϕ , we use a value of 1.0, which falls within the range considered in Hansen and Imrohoroglu (1992) and the existing RBC literature.

The distribution $F(\varepsilon)$ is assumed to be exponential. Specifically, $f(\varepsilon) = \frac{1}{\mu_\varepsilon} \exp\left(-\frac{\varepsilon}{\mu_\varepsilon}\right)$, so that $F(\varepsilon) = 1 - \exp\left(-\frac{\varepsilon}{\mu_\varepsilon}\right)$. We normalize the value $\mu_\varepsilon = 1$.

For the matching function, m , we use the standard constant returns to scale form given by $m(u, v) = u_j^\eta v^{1-\eta}$.¹⁴ As in Fredriksson and Holmund (2001), we use a value of 0.5 for β , and set $\eta = \beta$.

The job separation rate is set to match the average unemployment rate from 1989 – 2012, which was 6.0%. This implies a value of $\lambda = 0.0155$. This value of λ is consistent with the value in Shimer (2005), who finds a quarterly job separation rate of 0.035. An alternative calibration strategy is to

¹⁴An equivalent alternative, used by others including Shimer (2005), is $m(u, v) = m_0 u^\eta v^{1-\eta}$ where u/v is normalized to 1, and m_0 is chosen to target the number of matches.

fix λ at the rate in [Shimer \(2005\)](#) and let the unemployment rate differ from the average in the data. The value of γ (vacancy creation costs) is set to match the observed average unemployment duration from 1989 – 2012. According to the BLS tabulations from the CPS, the average unemployment duration from 1989 – 2012 was 18.1 weeks, or 4.53 months, implying $\gamma = 22.12$.

We parameterize the UI system setting $b = 0.45$, consistent with an average replacement rate of 0.50. For the minimum level of consumption (among UI non-collectors), we set d to match the observed take-up rate. Recall from [Proposition 3](#), the equilibrium value of ε^* , a key determinant of the take-up rate (from [Equation \(28\)](#)), is determined by $h(b) - h(d)$. For the 1989 – 2012 period, the take-up rate averaged 77%, requiring $d = 0.1672$.

Finally, the value of τ is set to match data on experience rating in the U.S. system. Specifically, we target the average value for the ERI described in [Section 2](#). The average ERI from 1989 – 2004 is 60.¹⁵ We interpret this as the average marginal cost of separating from a worker, in terms of increased UI taxes for the firm.¹⁶ In the model, the average worker who collects UI incurs benefit expenditures equal to the UI benefit, b , times the average duration of unemployment (since benefits do not expire). We set τ to be 60% of this average benefit expenditure, implying $\tau = 1.387$. [Table 1](#) lists the parameters and their values.

5.2 Results

[Table 2](#) presents the results from our calibration. [Figures 6\(a\) to 6\(d\)](#) show the key properties of equilibrium established in [Section 4](#). To begin, [Figures 6\(a\) and 6\(b\)](#) display how equilibrium queue lengths and wages behave. As in [Section 4](#), arrival rates for UI collectors are increasing in

¹⁵Recall from [Section 2.2](#) that the data are available from 1989 – 2004 and from 2008 – 2012; however, the calculations are different in the two periods, and thus not comparable. The ERI averaged 68.9 from 2008-2012, implying the average over the entire period (1989-2004, 2008-2012) is 61.7.

¹⁶[Topel \(1983\)](#) also calculates the marginal cost of a separation to a firm, finding an average of approximately 80%. We provide some sensitivity analysis with respect to τ below in [Section 5.3](#).

Table 1: Parameters

r	0.0033	Discount rate
ϕ	1.0	Coefficient of relative risk aversion
β	0.5	Bargaining parameter
η	0.5	Elasticity of matching function
λ	0.0155	Job separation rate
b	0.45	Replacement rate, UI, non-binding
d	0.1672	Minimum consumption rate
γ	22.12	Vacancy cost
τ	1.387	Experience rating parameter
μ_ε	1	Parameter of $F(y)$

Table 2: Calibration Results

Moment	Model	Data
Unemployment rate	6.0%	6.0%
Unemployment duration	4.53	4.53
Take-up rate	0.76	0.77

ε and wages decrease. Recall, as ε increases, the net gain from collecting UI benefits decreases so workers begin to search for lower wage jobs that arrive faster.

Figures 6(a) and 6(b) also display the relationship between UI collector and non-collector wages and queue lengths. As indicated in Corollary 1, UI non-collectors have shorter unemployment durations than UI collectors (consistent with the empirical evidence in Katz and Meyer (1990)) and they have lower wages relative to UI collectors. This is consistent with the intuition above for the change in UI collector search behavior. Recall, at $\varepsilon = \varepsilon^*$, $h(b) - \varepsilon^* = h(d)$. Since the net benefit of collecting is higher than $h(d)$ for all UI collectors, non-collectors prefer lower wage jobs arriving faster.

Proposition 3 also shows that non-collector wages and queue lengths are distorted from the efficient levels. Figures 6(c) and 6(d) display the effect of experience rating on non-collector search

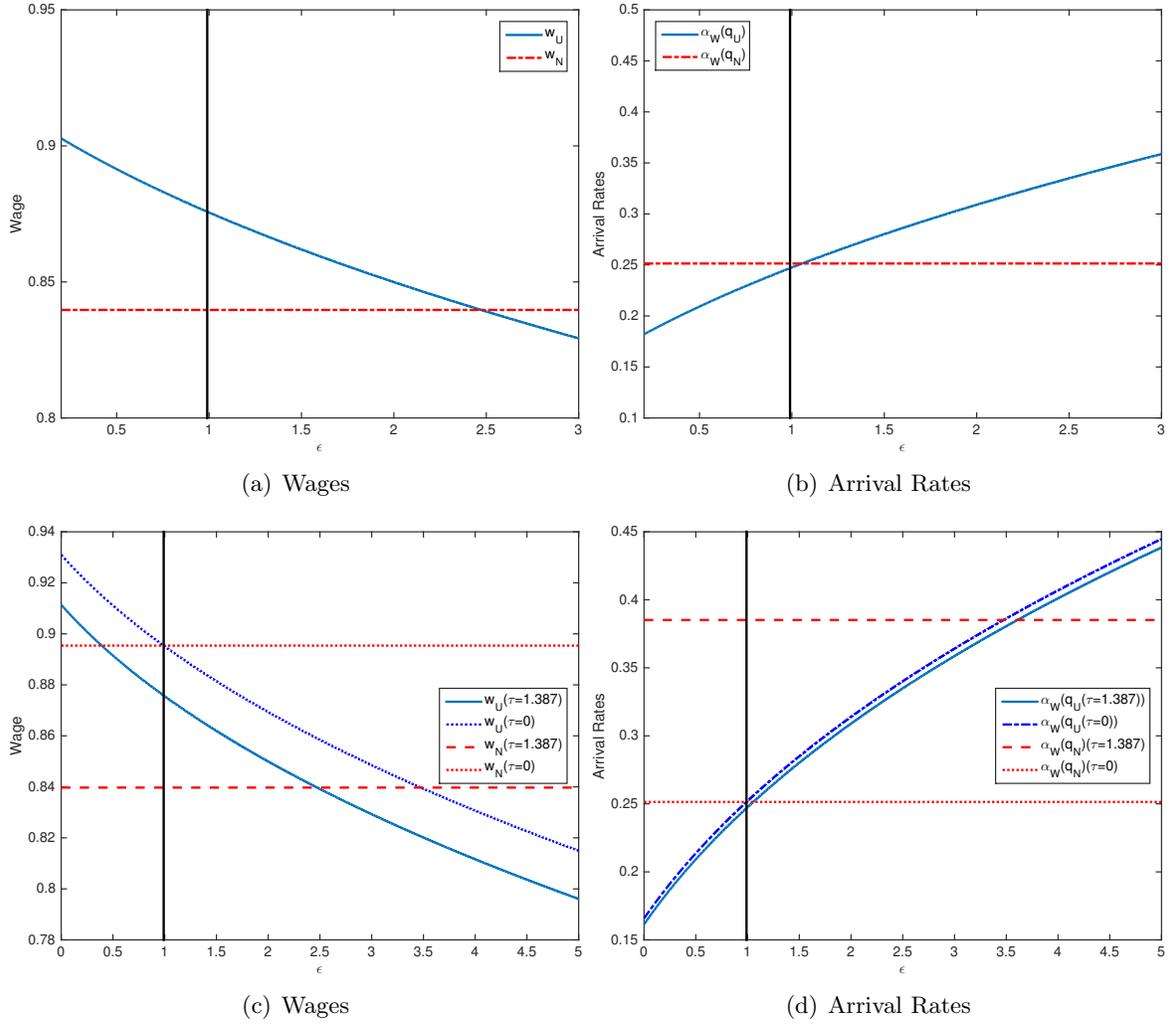


Figure 6: Wages and Arrival Rates Comparison

The upper-left figure plots equilibrium wages for UI collectors vs. non-collectors, as a function of ε . Similarly, the upper-right figure plots the respective arrival rates. In the lower two figures, we plot the effect of experience rating on wages and arrival rates. The lower-left figure plots equilibrium wages for UI collectors under the baseline $\tau = 1.387$ and when $\tau = 0$, as a function of ε . The lower-right figure plots the respective arrival rates.

behavior. Consider Figure 6(d), which plots job arrival rates for the baseline $\tau = 1.387$ and for $\tau = 0$. When $\tau = 0$, job arrival rates for non-collectors are lower than with $\tau = 1.387$. Table 3 shows the implications of this for the average duration of unemployment and average wages.

Examining the duration for non-collectors in Table 3 shows that the inefficiency in the equilibrium with experience rating reduces non-collectors' average duration of unemployment from 3.98 months with $\tau = 0$ to 2.6 months with $\tau = 1.387$. Recall that when $\tau = 0$, non-collector queue lengths and wages are efficient. Thus, non-collectors move to employment over one month (5.52 weeks) *sooner* than what is optimal, and their wage is 6.55% lower than the optimal level.

The empirical literature on UI benefits has found that UI non-collectors have shorter unemployment durations relative to an equivalent collector. Our model also implies this feature, as does the existing search literature. In our model, however, part of the difference in unemployment durations between collectors and non-collectors is inefficient, arising from the combination of an informational friction (firms do not observe past UI collections) and the experience rated UI tax scheme. UI non-collectors are better off with a longer average duration of unemployment and higher wage than what occurs in equilibrium.

Figures 6(c) and 6(d) also summarize the key implications of experience rated taxes on job arrival rates and wages for UI collectors. Consider first the case of $\tau = 0$. Notice that as described in Proposition 2, at $\varepsilon = \varepsilon^*$, $q_N^* = q_U^*(\varepsilon^*)$; as a result, non-collectors and collectors have the same wage and job arrival rate. For $\varepsilon \leq \varepsilon^*$, jobs arrive slower for UI collectors, but they search for higher wage jobs. When τ increases to the baseline $\tau = 1.387$, the job arrival rate and wage decrease for all UI collectors (for each $\varepsilon \leq \varepsilon^*$). The increase in τ implies a parallel shift down in the firm's zero-profit curve, as described by Equation (14). As a result, the UI-collector's indifference curve (for a given ε) is tangent further to the "right" on the zero-profit curve; q_u is higher, but w_u is

Table 3: Effect of Experience Rating: Wages and Arrival Rates

Moment	Baseline τ			$\tau = 0$		
	Collectors	Non-collectors	Ratio	Collectors	Non-collectors	Ratio
Duration	5.134	2.596	1.98	5.017	3.978	1.261
Wage	0.895	0.840	1.065	0.915	0.895	1.022

For UI collectors, the unemployment duration and wage vary with ε . The table reports the respective averages across ε . Specifically, $\bar{w}_U = \int_0^{\varepsilon^*} w_U(\varepsilon)\phi_E(\varepsilon)d\varepsilon$ and $\text{DUR} = \int_0^{\varepsilon^*} \frac{1}{\alpha_W[q_U(\varepsilon)]}\phi_U(\varepsilon)d\varepsilon$, where $\phi_i(\varepsilon) = \frac{n_U^i(\varepsilon)}{N_U^i}$.

lower via the increase in τ (recall Equation (14)).

Table 3 compares the outcomes for wages and unemployment durations in each case. For UI collectors, when $\tau = 0$ the average unemployment duration is 5.134 months, compared to 5.017 months in the baseline case (when $\tau = 1.387$). When the level of experience rating is zero, the average unemployment duration of UI collectors goes down by around one half of a week. In addition, the average wage of UI collectors increases by about 2%.

Of course, this experiment is somewhat misleading, as when $\tau = 0$ there is no UI tax at all. In the next section, we explore the welfare implications of the inefficiency in non-collector allocations under a more comparable reduction in experience rating.

5.3 Welfare

To further understand the implications of the take-up decision and experience rating, we now analyze the effect of several policy changes on welfare. The comparisons use the following welfare

function:

$$\begin{aligned}
H = & \int_0^{\varepsilon^*} n_U^E(\varepsilon)h[w_U(\varepsilon)]d\varepsilon + \int_0^{\varepsilon^*} n_U^U(\varepsilon)[h(b) - \varepsilon]d\varepsilon - \gamma \left[\int_0^{\varepsilon^*} \frac{n_U^E(\varepsilon)}{q_U(\varepsilon)}d\varepsilon \right] \\
& + N_N^E h(w_N) + N_N^U h(d) - \gamma \frac{N_N^E}{q_N}
\end{aligned} \tag{30}$$

The first term in equation (30) is the total flow utility for employed workers who are UI collectors (when unemployed) and the second term is the analogous flow utility term for unemployed UI collectors. This total flow utility from UI collectors (both employed and unemployed) is taken net of the vacancy creation costs for firms employing these workers, which is the last term on the first line of equation (30). The second line of the equation above contains the same terms for those non-UI collecting workers; that is, weighted flow utility net of vacancy creation costs.

Table 4 summarizes the key welfare results. First consider the results presented in the second column labeled $\tau = 1.387$ (each column refers to a different level of experience rating). The 0.6 in parentheses refers to the fraction of expected benefits financed by τ ; i.e. $\frac{\tau}{b * DUR_U}$. The row for N gives the % consumption equivalent welfare gain of moving from the equilibrium \tilde{q}_N to the optimal one, q_N^* , for unemployed non-collectors. Given log utility, the % consumption equivalent is calculated as $100 \left(\exp \left(r(N(q_N^*) - N(\tilde{q}_N)) \right) - 1 \right)$. In the baseline case, the consumption equivalent welfare gain for unemployed non-collectors is relatively small at 0.008%.

The row labeled W_N displays the consumption equivalent welfare gain of moving from \tilde{q}_N to q_N^* for employed non-collectors (calculated as $100 \left(\exp \left(r(W_N(q_N^*) - W_N(\tilde{q}_N)) \right) - 1 \right)$). For $\tau = 1.387$, the welfare loss for employed non-collectors is 1.15%. As τ increases from partial experience rating to full experience rating (100% of the cost of benefit expenditures is charged to the firm), the welfare loss increases for both unemployed and employed non-collectors. Under full experience

rating, $\tau = 2.32$, the welfare loss to employed non-collectors is 1.60%.

Table 4: % Consumption-Equivalent Welfare Change

Utility	Level of Experience Rating, τ			
	1.387 (0.6)	1.70 (0.73)	2.0 (0.86)	2.32 (1.00)
N	0.008	0.009	0.011	0.013
W_N	1.15	1.31	1.45	1.60
H	2.82	3.27	3.69	4.12

Each cell contains the value of flow utility for each component of welfare under different levels of experience rating.

The last row, labeled H , uses the welfare function in Equation (30) to compute the consumption equivalent welfare gain relative to an alternative tax scheme. As discussed in Proposition 2, $\tilde{q}_N = q_N^*$ when $\tau = 0$. Of course, if $\tau = 0$, welfare in equilibrium increases; however, some of this increase arises simply because the unemployment benefits are completely subsidized (*i.e.* not paid for). Recall, $\tilde{q}_N = q_N^*$ is also achieved if all firms pay the tax τ , regardless of whether a separated employee collects UI or not. Thus, in the row labeled H , we compare the economy with experience rating to an economy with no experience rating (all firms pay the same tax), but where the tax schemes raise the same amount of revenue. Let $\hat{\tau}$ denote the no-experience rating tax. We set this according to:

$$\hat{\tau} = \frac{\tau N_U^E}{1 - \hat{u}}$$

where N_U^E is the number of employed UI collectors under τ , and \hat{u} is the unemployment rate in the economy under $\hat{\tau}$. Note, both the numerator and denominator are multiplied by λ , since the tax is paid only in the event of a separation, but these cancel since the separation rate is exogenous. Since under $\hat{\tau}$ the tax is spread across all employed workers, $\hat{\tau} < \tau$. For example, under $\tau = 1.387$,

$\hat{\tau} = 0.8665$ raises equivalent revenue. In addition, since H in Equation (30) is already in flow utility, the welfare gain is calculated as $100 * (\exp(H(\hat{\tau}) - H(\tau)) - 1)$.

In Table 4, under the baseline level of experience rating ($\tau = 1.387$), moving to no experience rating yields a welfare gain of 2.82% in consumption equivalent terms. As the level of experience rating increases, the gains to moving to a no-experience rated system increase, reaching as high as a 4.12% gain when $\tau = 2.32$ (perfect, or full, experience rating). These gains arise from two sources. First, under $\hat{\tau}$ the equilibrium is efficient, *i.e.* $\tilde{q}_N = q_N^*$. Although non-collectors now pay a tax while employed, reducing wages, this loss is more than offset by the efficiency gain in (q_N, w_N) . Second, workers who are UI collectors experience an increase in utility as the tax decreases (recall $\hat{\tau} < \tau$).

5.4 Discussion of Experience Rating

It is interesting to note that in this model experience rating has unambiguously negative effects on equilibrium outcomes. This is in contrast to the existing literature examining the effects of experience rating (examples include Feldstein (1976), Topel (1983), Wang and Williamson (2002), and Cahuc and Malherbet (2004)). The existing literature focuses on the effects of experience rating on job separations and firm decisions whether to layoff a worker or simply reduce their hours worked. Generally, the results suggest that a move from partial to full experience rating improves outcomes and welfare. These benefits accrue by generating a more efficient level of unemployment, which is lower under full experience rating.

Our model has implications for the impact of experience rating on the search side of the labor market. Here, we find that moving from full to no experience rating is optimal. As with the literature focusing on the separation effects of experience rating, our model implies that full experience

rating is associated with a lower unemployment rate; however, this lower unemployment rate is inefficient. With experience rating, $\tilde{q}_N < q_N^*$, and this gap increases as the level of experience rating increases. While UI collectors do experience a longer unemployment duration as τ increases (see Figure 6(d)), it is dominated by the effect on non-collectors. Although out of the scope of this paper, an interesting direction for future research is to re-consider the implications of experience rating for the separation decision when the take-up decision is also endogenous.

5.5 Effects of UI Benefits

In this section we consider the equilibrium impact of changing the level of unemployment benefits.¹⁷ The economy displays several interesting features when UI benefits increase. In the experiments below, we increase the UI replacement rate while setting τ to maintain the initial level of experience rating. That is, we maintain $\frac{\tau}{b * DUR_U} = 0.60$ as the replacement rate changes.

Figures 7(a) to 7(d) display the effects of an increase in b on the key equilibrium outcomes. First consider Figure 7(a). The take-up rate is increasing and concave in the UI replacement rate. It increases, from a low of 0.083 to a maximum of 0.835. Next, in Figure 7(b), the unemployment rate and average duration of unemployment are also increasing in the UI replacement rate. While indeed both moments increase with the replacement rate, they do so relatively slowly. A replacement rate of 100% is associated with an unemployment rate of 8.33%. Typically the unemployment rate and duration explode as the replacement rate approaches 100%. To understand the significance of these results, we compare them to a simple economy with a fixed 100% take-up rate.

Consider a standard directed search model with no UI collection costs (*i.e.* $\varepsilon = 0$ for all workers) and assume that all unemployed workers collect UI benefits. Thus, the take-up rate is fixed at 100%.

¹⁷Davidson and Woodbury (1998) and Wang and Williamson (2002) also examine unemployment insurance policies in models with take-up rates less than 1, but in these papers the take-up rate is exogenous.

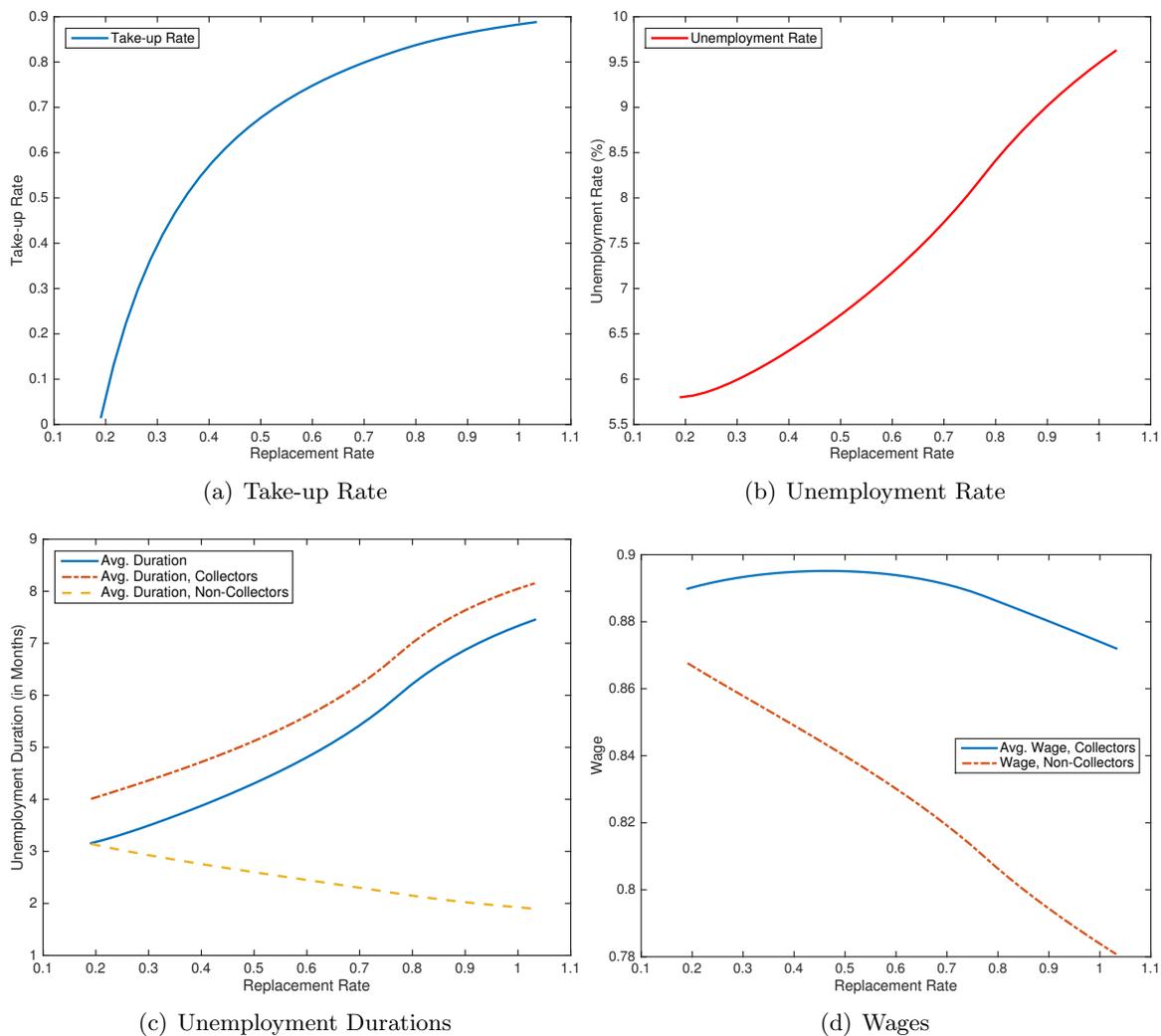


Figure 7: Effects of UI Benefits

This figure plots the effects of UI benefits on equilibrium outcomes. The top two graphs plots the Take-up and Unemployment rates, respectively. The bottom-left graph plots the effect of b on unemployment durations. It plots the overall average unemployment duration, as well as for collectors and non-collectors separately. Similarly, the bottom-right graph plots wages for collectors and non-collectors. In all figures, the horizontal axis corresponds to the average replacement ratio for that particular b , or $\frac{b}{\bar{w}_U}$, where $\bar{w}_U = \int_0^{\varepsilon^*} w_U(\varepsilon)\phi_E(\varepsilon)d\varepsilon$ is the average wage for UI collectors.

We simulated the economy under different replacement rates and report the results in Figures 8(a) to 8(d). In these simulations, we maintain the same parametrization as the baseline case (with the exception that $\varepsilon = 0$).

Figure 8(a) and Figure 8(b) plot the response of the unemployment rate and average duration of unemployment, respectively. As one typically expects, as the replacement rate approaches 100%, both of the aforementioned moments begin to explode. At a replacement rate of 86% the unemployment rate is 19.12% and the average unemployment duration is 16.18 months. This is a significantly faster response relative to our baseline model, which is plotted in Figure 8(c) and Figure 8(d), respectively. Indeed, the endogenous take-up rate represents the key difference between the two models.

The difference in responses of the unemployment rate and average duration of unemployment to UI benefits derives in part from the following. As UI benefits increase, the average duration of unemployment for UI collectors indeed increases; however, it *decreases* for UI non-collectors. Moreover, the take-up rate increases at a decreasing rate with UI benefits. Put together, the average duration of unemployment (economy-wide), which is the average between collectors and non-collectors), eventually levels off similarly to the take-up rate.

6 Conclusion

We estimate the UI take-up rate for the U.S. economy from 1989 – 2012. An equilibrium directed search model is developed to explain this empirical fact and explore its implications for the provision of UI benefits. Modeling the take-up decision leads to an informational friction that creates an inefficiency in non-collector outcomes.

Consistent with empirical studies on the effects of UI benefits, the model predicts that UI

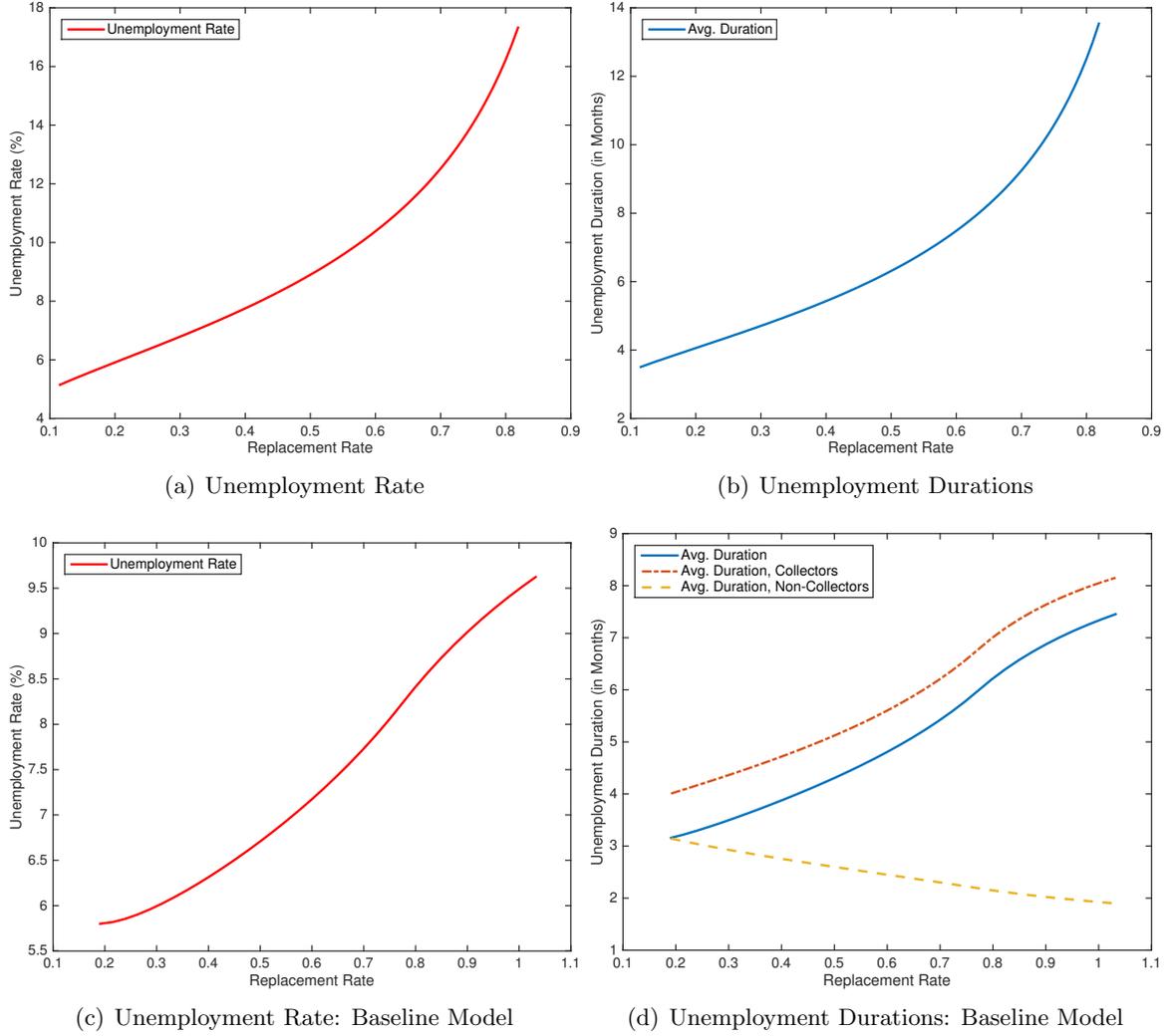


Figure 8: Effects of UI Benefits: 100% Take-up Model

This figure plots the effects of UI benefits on equilibrium outcomes for a standard model with 100% take-up. The top two graphs plot the responses of the Unemployment Rate and Average Unemployment Duration for the 100% take-up rate economy, respectively. For comparison, the bottom-left graph plots the effect of b on the Unemployment Rate in the baseline economy, and the bottom-right graph plots Unemployment Durations in the baseline economy. In all figures, the horizontal axis corresponds to the average replacement ratio for that particular b , or $\frac{b}{w_U}$.

collectors have longer unemployment durations than non-collectors. Part of this difference is inefficient, as non-collectors transition to employment faster than the optimal rate. After calibrating the model, we explore several counterfactual policy experiments. We find that the inefficiency imposed by the presence of non-collectors and experience rated taxes amounts to a welfare cost of 2.82%. This welfare gain can be achieved when all firms are taxed equally. Finally, we also show that incorporating the take-up decision matters when examining the effects of UI benefits on equilibrium outcomes. The analysis indicates that the unemployment rate and average duration of unemployment respond slower to changes in UI benefits than the standard search model with a fixed 100% take-up rate.

Our analysis also show how experience rating of firm UI taxes affects the hiring decisions of firms, and we find full experience rating reduces welfare. It remains possible, however, that such experience rating also affects firm layoff decisions. [Feldstein \(1976\)](#), [Topel \(1983\)](#), and [Cahuc and Malherbet \(2004\)](#), among others, examine how the partial experience rating in the U.S. affects firm separation decisions and find that full experience rating improves welfare. While in our model we assume separations are exogenous, incorporating this dimension into the analysis of take-up rates represents an interesting direction for future research.

References

- ACEMOGLU, D., AND R. SHIMER (1999): “Efficient Unemployment Insurance,” *Journal of Political Economy*, 107, 893–928.
- (2000): “Productivity Gains from Unemployment Insurance,” *European Economic Review*, 44, 1195–1224.
- ALBRECHT, J., AND S. VROMAN (1999): “Unemployment Finance and Efficiency Wages,” *Journal of Labor Economics*, 17, 141–167.
- ANDERSON, P., AND B. MEYER (1997): “Unemployment Insurance Takeup Rates and the After-Tax Value of Benefits,” *The Quarterly Journal of Economics*, 112, 913–937.
- AURAY, S., AND D. L. FULLER (2016): “Eligibility, Generosity, and Take-up: A State Level Analysis of Unemployment Insurance in the U.S.,” Working paper, CREST-ENSAI.
- BLANK, R., AND D. CARD (1991): “Recent Trends in Insured and Uninsured Unemployment: Is There an Explanation?,” *The Quarterly Journal of Economics*, 106, 1157–1189.
- CAHUC, P., AND F. MALHERBET (2004): “Unemployment Compensation Finance and Labor Market Rigidity,” *Journal of Public Economics*, 88, 481–501.
- DAVIDSON, C., AND S. WOODBURY (1998): “The optimal dole with risk aversion and job destruction,” Working paper, Michigan State University.
- FELDSTEIN, M. (1976): “Temporary Layoffs in the Theory of Unemployment,” *Journal of Political Economy*, 84, 937–958.

- FREDRIKSSON, P., AND B. HOLMUND (2001): “Optimal Unemployment Insurance in Search Equilibrium,” *Journal of Labor Economics*, 19, 370–399.
- FULLER, D. L., B. RAVIKUMAR, AND Y. ZHANG (2015): “Unemployment Insurance Fraud and Optimal Monitoring,” *American Economic Journal: Macroeconomics*, 7, 249–290.
- HANSEN, G., AND A. IMROHOROGLU (1992): “The role of unemployment insurance in an economy with liquidity constraints and moral hazard,” *Journal of Political Economy*, 100, 118–142.
- KATZ, L. F., AND B. D. MEYER (1990): “The Impact of the Potential Duration of Unemployment Benefits on the Duration of Unemployment,” *Journal of Public Economics*, 41, 45–72.
- MARIMON, R., AND F. ZILIBOTTI (1999): “Unemployment vs. Mismatch of Talents: Reconsidering Unemployment Benefits,” *The Economic Journal*, 109, 266–291.
- MOEN, E. R. (1997): “Competitive Search Equilibrium,” *Journal of Political Economy*, 105, 385–411.
- NAKAJIMA, M. (2012): “A Quantitative Analysis of Unemployment Benefit Extensions,” *Journal of Monetary Economics*, 59, 686–702.
- ROGERSON, R., R. SHIMER, AND R. WRIGHT (2005): “Search Theoretic Models of the Labor Market: A Survey,” *Journal of Economic Literature*, 43, 959–988.
- SHIMER, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95, 25–49.
- TOPEL, R. H. (1983): “On Layoffs and Unemployment Insurance,” *American Economic Review*, 83, 541–559.

WANG, C., AND S. WILLIAMSON (2002): “Moral hazard, optimal unemployment insurance, and experience rating,” *Journal of Monetary Economics*, 49, 1337–1371.

A Proofs

The following Lemma is used in the Proof of Proposition 1.

Lemma 1 *The value function, $U(\varepsilon)$ is strictly decreasing in ε ; i.e. $\frac{\partial U(\varepsilon)}{\partial \varepsilon} < 0$.*

Proof: This follows immediately from Equation (14) and the envelope theorem. ■

Proof of Proposition 1:

Proof: To prove Proposition 1, define the function $\Gamma(\varepsilon) = U(\varepsilon) - N$. Now consider $\Gamma(0)$. If $\Gamma(0) > 0$, the functions never cross (implying the take-up rate is 0). Consider then the case where $\Gamma(0) \leq 0$. Here we show that there is a unique crossing point. Since $U(\infty) = -\infty$, we also have that $\Gamma(\infty) = -\infty$. Finally, from Lemma 1, $\Gamma(\varepsilon)$ is strictly decreasing (and continuous); therefore, there exists a unique ε^* such that $U(\varepsilon^*) = N$. ■

For the remaining proofs, it is useful to work with the worker's indifference curve. This is derived using Equation (6). Specifically, for any level of utility \bar{U} , an unemployed UI collector's indifference curve is given by:

$$\mathcal{W}(q) = h^{-1} \left\{ \frac{1}{\alpha_W(q)} \left[\bar{U}(r + \lambda + \alpha_W(q)) - (r + \lambda)[h(b) - \varepsilon] \right] \right\} \quad (31)$$

To ease the notation, define $T(q)$ as:

$$T(q) = \frac{1}{\alpha_W(q)} \left[\bar{U}(r + \lambda + \alpha_W(q)) - (r + \lambda)[h(b) - \varepsilon] \right] \quad (32)$$

Given this, we have

$$\frac{\partial \mathcal{W}(q)}{\partial q} = \frac{T'(q)}{h'(h^{-1}(T(q)))} \quad (33)$$

where

$$T'(q) = \frac{-\alpha'_W(q)}{[\alpha_W(q)]^2} (\bar{U} - [h(b) - \varepsilon]) \quad (34)$$

Note that in equilibrium, since we restrict attention to $q(\varepsilon)$ such that $w(q) \geq \max\{h(b) - \varepsilon, h(d)\}$ (depending on whether or not the worker collects UI), $\bar{U} \geq h(b) - \varepsilon$; as a result, since $\alpha'_W(q) < 0$, $T'(q) > 0$. That is, the worker's indifference curve is strictly increasing in (q, w) space. Related, define the zero profit function defined in Equation (14) as:

$$\mathcal{P}(q) = y - \frac{\gamma(r + \lambda)}{q_U(\varepsilon)\alpha_W(q_U(\varepsilon))} - \lambda\tau \quad (35)$$

Viewed in this way, the problem of determining the optimal q becomes one of finding the indifference curve tangent to the firm's zero profit curve. The next Lemma shows that $\mathcal{P}(q)$ is strictly increasing and strictly concave.

Lemma 2 *The wage defined in Equation (35) (and Equation (14)) is such that $\mathcal{P}'(q) > 0$ and $\mathcal{P}''(q) < 0$.*

Proof: First, recall that our matching function is assumed to be such that $\alpha'_E(q) > 0$ and $\alpha''_E(q) < 0$. Differentiating Equation (35) with respect to q gives,

$$\mathcal{P}'(q) = \frac{\gamma(r + \lambda)\alpha'_E(q)}{[\alpha_E(q)]^2}$$

which is > 0 given the properties of $\alpha_E(q)$. Differentiating again with respect to q yields,

$$\mathcal{P}''(q) = \frac{\gamma(r + \lambda)\alpha''_E(q)[\alpha_E(q)]^2 - 2\gamma(r + \lambda)\alpha'_E(q)\alpha_E(q)}{[\alpha_E(q)]^2}$$

which is < 0 since $\alpha''_E(q) < 0$ and $\alpha'_E(q) > 0$. ■

The following Lemma is used in several proofs below:

Lemma 3 *The job arrival rate $\alpha_W[q_U(\varepsilon)]$ is increasing in ε and the wage $w_U(\varepsilon)$ is decreasing in ε .*

Proof: Consider ε_1 and ε_2 such that $\varepsilon_2 > \varepsilon_1$. Denote $q_U^*(\varepsilon)$ the optimal choice of queue length and $U^*(\varepsilon)$ the associated indirect utility (defined by Equation (13)) for a given ε . Since $\alpha_W(q)$ is strictly decreasing in q , we need to show that $q_U^*(\varepsilon_1) > q_U^*(\varepsilon_2)$. Now, suppose instead that $q_U^*(\varepsilon_2) \geq q_U^*(\varepsilon_1)$. From Lemma 1, $U^*(\varepsilon_1) > U^*(\varepsilon_2)$. Notice then, for a given q , $T'(q; \varepsilon_1) > T'(q; \varepsilon_2)$, as $T'(q)$ is increasing in \bar{U} (from Equation (34)). Since by assumption, $q_U^*(\varepsilon_2) \geq q_U^*(\varepsilon_1)$ and $T(q)$ is increasing we also have that $T'(q_U^*(\varepsilon_1)) > T'(q_U^*(\varepsilon_2))$ given $T'(q)$ is increasing in q (see above).

Moreover, from the properties of the utility function, $h''(w) < 0$, so that $h'(w)$ is decreasing. Thus,

$$\frac{1}{h'(h^{-1}(w_U^*(\varepsilon_1)))} > \frac{1}{h'(h^{-1}(w_U^*(\varepsilon_2)))}, \text{ implying that } \frac{T'(q_U^*(\varepsilon_1))}{h'(h^{-1}(w_U^*(\varepsilon_1)))} > \frac{T'(q_U^*(\varepsilon_2))}{h'(h^{-1}(w_U^*(\varepsilon_2)))}.$$

Now, by definition of being an optimal solution, $q_U^*(\varepsilon_1)$ satisfies,

$$\mathcal{P}'(q_U^*(\varepsilon_1)) = \frac{\partial \mathcal{W}}{\partial q} = \frac{T'(q_U^*(\varepsilon_1))}{h'(h^{-1}(w_U^*(\varepsilon_1)))}$$

Since $\mathcal{P}''(q) < 0$, $\mathcal{P}'(q_U^*(\varepsilon_2)) > \mathcal{P}'(q_U^*(\varepsilon_1))$, which combined with the results above implies,

$$\mathcal{P}'(q_U^*(\varepsilon_2)) > \mathcal{P}'(q_U^*(\varepsilon_1)) = \frac{T'(q_U^*(\varepsilon_1))}{h'(h^{-1}(w_U^*(\varepsilon_1)))} > \frac{T'(q_U^*(\varepsilon_2))}{h'(h^{-1}(w_U^*(\varepsilon_2)))}$$

which is a contradiction to $q_U^*(\varepsilon_2)$ being an optimal solution to Equation (13). Therefore, $q_U^*(\varepsilon_2) < q_U^*(\varepsilon_1)$. ■

Lemma 3 describes how the equilibrium allocations to UI collectors depend on the direct utility cost of collecting benefits. Intuitively, as ε increases, net benefit provided by UI is reduced which acts similarly to a decrease in UI benefits. Hence, the worker prefers to trade-off lower wages for a faster job arrival rate.

Proof of Proposition 2:

Proof: To prove that equilibrium is efficient: When $\tau = 0$, $w_U(q) = w_N(q)$ (for the same q) and thus any choice of q_N, w_N is a feasible choice for $q_U(\varepsilon), w_U(\varepsilon)$ and vice versa. Thus, by definition of $U^*(\varepsilon)$ in Equation (13), $U^*(\varepsilon) \geq \tilde{U}(\varepsilon)$ for all ε . Importantly, the incentive constraint is satisfied for *any* choice of q_N . With no constraint to distort allocations, the equilibrium queue length for non-collectors is determined by Equation (15); therefore, $\tilde{q}_N = q_N^*$. Equilibrium is thus efficient, as firms maximize profits and lifetime expected utility is maximized for all workers.

Uniqueness of the equilibrium stems from the fact that the worker's indifference curve is strictly convex and the zero-profit curve is strictly concave, ensuring a unique maximum exists in Equations (13) and (15). In addition, there exists a unique solution to the flow equations in Equations (22) to (27) determining unique values of $N_i^E, N_i^u, i = U, N$. The uniqueness of ε^* from Proposition 1 thus implies the equilibrium with $\tau = 0$ is unique.

Next to show (i) $\varepsilon^* = h(b) - h(d)$. Define $\tilde{\varepsilon} = h(b) - h(d)$ and ε^* as such that $U(\varepsilon^*) = N$. Recall that when $\tau = 0$, q_N is defined by Equation (15) and $q_U(\tilde{\varepsilon})$ by Equation (13). Since $h(d) = h(b) - \tilde{\varepsilon}$, notice that Equations (13) and (15) solve the same problem; as a result, $q_N = q_U(\tilde{\varepsilon})$ and $w_N = w_U(\tilde{\varepsilon})$. Notice then, $U(\tilde{\varepsilon}) = N$. From Proposition 1, $U(\varepsilon)$ and N have a unique crossing point; therefore, $\varepsilon^* = \tilde{\varepsilon} = h(b) - h(d)$.

To show (ii): First, as shown above, at $\varepsilon = \varepsilon^*$, $h(b) - \varepsilon^* = h(d)$, implying that $U(\varepsilon^*) = N$, so $q_U^*(\varepsilon^*)$ and q_N^* solve the same problem; therefore, $q_U^*(\varepsilon^*) = q_N^*$. Then, from Lemma 3, $q_U^*(\varepsilon)$ is

strictly decreasing in ε , implying that $q_U^*(\varepsilon) < q_U^*(\varepsilon^*) = q_N^*$, $\forall \varepsilon < \varepsilon^*$.

Finally, to show (iii): At $\varepsilon = \varepsilon^*$, since $q_U^*(\varepsilon^*) = q_N^*$, from Equation (14) with $\tau = 0$ and Equation (16), $w_U^*(\varepsilon^*) = w_N^*$. Moreover, from Lemma 3 the wage is strictly decreasing in ε implying that $w_U^*(\varepsilon) > w_U^*(\varepsilon^*) = w_N^*$, $\forall \varepsilon < \varepsilon^*$. ■

The following Lemma is used in the proof of Proposition 3:

Lemma 4 *The value function $U(\varepsilon; \tau)$ is decreasing in τ .*

Proof: Differentiating $U(\varepsilon)$ in Equation (6) with respect to τ (using the Envelope Theorem) gives:

$$\frac{\partial U}{\partial \tau} = \left(\frac{\alpha_W(q_U(\varepsilon))}{r + \lambda + \alpha_W(q_U(\varepsilon))} \right) \left(h'(w_U(\varepsilon)) \right) \left(\frac{\partial w_U}{\partial \tau} \right)$$

The first two terms in parenthesis are positive, and from Equation (14), $\frac{\partial w_U}{\partial \tau} < 0$. ■

Proof of Proposition 3:

Proof: We begin with property (i), that $\tilde{q}_N \neq q_N^*$. Define $\tilde{\varepsilon}$ such that $U(\tilde{\varepsilon}) = N(\tilde{q}_N)$, where $N(q)$ is given by Equation (7). This simply represents the unique crossing point of U and N identified in Proposition 1. To show property (i), it is sufficient to show that at q_N^* (the solution to Equation (15)) the constraint is violated at $\varepsilon = \tilde{\varepsilon}$. That is, $\tilde{U}(\tilde{\varepsilon}) > U(\tilde{\varepsilon})$.

Recall, from Proposition 2 when $\tau = 0$, $\varepsilon^* = h(b) - h(d)$, $q_U(\varepsilon^*) = q_N^*$, and by definition, $U(\varepsilon^*) = N^*$. Now, from Lemma 4, $U(\varepsilon^*; \tau = 0) > U(\varepsilon^*; \tau) > 0$. As a result, for $\tau > 0$, at $\varepsilon^* = h(b) - h(d)$ the following must hold:

$$U(\varepsilon^*; \tau > 0) < U(\varepsilon^*; \tau = 0) = N^* = \tilde{U}(\varepsilon^*)$$

where $\tilde{U}(\varepsilon^*)$ is defined by Equation (17) evaluated at q_N^* . Notice, $\tilde{U}(\varepsilon)$ does not depend on τ , only q_N . Therefore, at q_N^* , the constraint in Equation (18) is violated at $\varepsilon = \varepsilon^*$. Now, $\tilde{U}(\varepsilon^*) > U(\varepsilon^*)$ does not matter if the crossing point of U and N occurs before, *i.e.* $\tilde{\varepsilon} < \varepsilon^*$, since these workers prefer not to collect UI benefits. If $\tilde{\varepsilon} < \varepsilon^*$, then

$$N(\tilde{q}_N) = U(\tilde{\varepsilon}) > U(\varepsilon^*) = N^*(q_N^*)$$

where the inequality follows from $U(\varepsilon)$ being decreasing in ε (see Lemma 1). This is a contradiction, however, to N^* solving Equation (15). Therefore, $\tilde{q}_N \neq q_N^*$.

To show property (ii) we work with the constraint in Equation (18) at $\tilde{\varepsilon}$ (the crossing point of U and $N(\tilde{q}_N)$). Towards this end, first notice that since $\varepsilon^* = h(b) - h(d)$, it must be that $N(\tilde{q}_N) = \tilde{U}(\varepsilon^*)$ (since both value functions have the same flow utility at $\varepsilon = \varepsilon^*$). As a result, according to Equation (18), we must have:

$$U(\varepsilon^*) \geq \tilde{U}(\varepsilon^*) = N(\tilde{q}_N) \tag{36}$$

If it binds, we are done as by definition $\tilde{\varepsilon} = \varepsilon^*$. Suppose instead that $U(\varepsilon^*) > N = U(\tilde{\varepsilon})$, which from Lemma 1 implies that $\tilde{\varepsilon} > \varepsilon^*$. This is not possible, however, as a firm could set q_N such that $N(q_N) = U(\varepsilon^*) > N$, a contradiction to \tilde{q}_N being an equilibrium (as no one would apply to that job). As a result, Equation (18) must bind at $\tilde{\varepsilon}$, the U and N crossing point.

Finally, to show property (iii), we need to show that in the notation above, $\tilde{\varepsilon} = \varepsilon^* \equiv h(b) - h(d)$. Suppose instead $\tilde{\varepsilon} \neq \varepsilon^* \equiv h(b) - h(d)$. Then there are two possible cases: (i) $\tilde{\varepsilon} < \varepsilon^*$ and (ii) $\tilde{\varepsilon} > \varepsilon^*$.

First, recall that by definition of $\varepsilon^* = h(b) - h(d)$, $\tilde{U}(\varepsilon^*) = N$. In Case (i), if $\tilde{\varepsilon} < \varepsilon^*$, then

$$\tilde{U}(\varepsilon^*) = N = U(\tilde{\varepsilon}) < U(\varepsilon^*)$$

where the last inequality follows from Lemma 1. This implies a contradiction, however, as Equation (18) is violated for UI collectors. In Case (ii), if $\tilde{\varepsilon} > \varepsilon^*$, then Lemma 1 implies

$$N = U(\tilde{\varepsilon}) < U(\varepsilon^*)$$

This cannot hold in equilibrium, however, since a firm could set q_N such that $N = U(\varepsilon^*)$, which increases non-collector utility. As a result, all non-collectors would apply to this job, a contradiction.

Therefore, $\tilde{\varepsilon} = \varepsilon^* = h(b) - h(d)$. ■

Proof of Corollary 1:

Proof: For notation, let $U^* \equiv U(q_U^*(\varepsilon^*); \tau)$ denote the equilibrium lifetime utility delivered to an unemployed UI collector with $\varepsilon = \varepsilon^*$, for $\tau > 0$. Similarly, let $\tilde{N} \equiv N(\tilde{q}_N)$ denote the equilibrium lifetime utility delivered to an unemployed UI non-collector when $\tau > 0$ and $N^* \equiv N(q_N^*)$ the optimal solution achieved under $\tau = 0$. Furthermore, denote the firm's zero profit curve from Equation (35) for workers collecting UI (with τ paid at separation) by $\mathcal{P}_U(q)$ and for a non-collector as $\mathcal{P}_N(q)$. Note, given $\tau > 0$, Equation (35) implies that given any q , $\mathcal{P}_N(q) > \mathcal{P}_U(q)$.

In equilibrium, the “incentive” constraint imposed by Equation (21) implies that a non-collector and a UI collector with $\varepsilon = \varepsilon^*$ must be on the same indifference curve. That is, $\mathcal{W}(\tilde{q}_N) = \mathcal{W}(q_U^*(\varepsilon^*))$. In addition, equilibrium requires the zero profit curve to intersect the indifference curve at the equilibrium q . For UI collectors this is a tangency, while for non-collectors (when $\tau > 0$) it is an intersection, which we show happens twice. To show this, we start by showing that at $q = q_N^*$,

$\mathcal{W}(q) - \mathcal{P}_N(q) < 0$ and crosses zero twice, once with $\tilde{q}_N^L < q_N^*$ and once with $\tilde{q}_N^H > q_N^*$.

From Lemma 4 and Proposition 2, $N^* = U(q_U^*(\varepsilon^*); \tau = 0) > U(q_U^*(\varepsilon^*); \tau) = \tilde{N}$. As a result, $\mathcal{P}_N(q_N^*) = \mathcal{W}(q_N^*; N^*) > \mathcal{W}(\tilde{q}_N; \tilde{N}) = \mathcal{W}(q_U^*(\varepsilon^*); U^*) = \mathcal{P}_U(q_U^*(\varepsilon^*))$. Thus, at q_N^* , $\mathcal{W}(q_N^*) - \mathcal{P}_N(q_N^*) < 0$. Now, consider $\mathcal{W}(q) - \mathcal{P}_N(q)$ as q decreases. Towards this end, given the properties of the matching function, notice that $\lim_{q \rightarrow 0} \alpha_W(q) = \infty$, $\lim_{q \rightarrow \infty} \alpha_W(q) = 0$, $\lim_{q \rightarrow 0} \alpha_E(q) = 0$, and $\lim_{q \rightarrow \infty} \alpha_E(q) = \infty$.

$$\lim_{q \rightarrow 0} \mathcal{W}(q) = U^* - (r + \lambda)h(d) \quad (37)$$

$$\lim_{q \rightarrow \infty} \mathcal{W}(q) = h^{-1}[\infty] = \infty \quad (38)$$

$$\lim_{q \rightarrow 0} \mathcal{P}(q) = -\infty \quad (39)$$

$$\lim_{q \rightarrow \infty} \mathcal{P}(q) = y - \chi_i \lambda \tau \quad (40)$$

where recall $\chi_i, i = U, N$ is an indicator variable with $\chi_U = 1$ and $\chi_N = 0$. Using Equations (37) to (40) implies that $\lim_{q \rightarrow 0} \mathcal{W}(q) - \mathcal{P}_N(q) > 0$. Thus, it starts negative at $q = q_N^*$ and is eventually positive. Since $\mathcal{W}(q)$ is strictly convex (and strictly increasing) and $\mathcal{P}(q)$ is strictly concave (and strictly increasing), this crossing only happens once. As a result, there exists an equilibrium $\tilde{q}_N^L < q_N^*$. We similarly show that there exists an equilibrium $\tilde{q}_N^H > q_N^*$. Specifically, Equations (37) to (40) imply that $\lim_{q \rightarrow \infty} \mathcal{W}(q) - \mathcal{P}_N(q) > 0$, which combined with the strict convexity of $\mathcal{W}(q)$ and strict concavity of $\mathcal{P}(q)$ yields a unique crossing above q_N^* .

B Experience Rating Index

This section discusses the details of experience rating system in the U.S. UI system. Actual determination of a firm's tax rate depends on the particular rules and method employed by that U.S. state. Thus, the extent of experience rating and how a firm's tax rate responds to changes in its experience with insured unemployment varies significantly across states. A detailed analysis of these tax schemes is beyond the scope of this paper, but this section describes some essential characteristics that illuminate the extent of state-level variations.

Each state has a particular formula for calculating a firm's tax rate. These formulas fall into four broad categories: 1) Reserve ratio, 2) Benefit ratio, 3) Benefit wage ratio, and 4) Payroll decline. The first two are the most popular, and the last, Payroll decline is utilized only in Alaska. To gain an understanding of the calculations, consider the reserve ratio. A firm's reserve ratio is calculated as,

$$\text{Reserve Ratio} = \frac{\text{Contributions-Benefits Charged}}{\text{Payroll}}$$

Here *Contributions* refers to taxes already collected from the firm, *Benefits charged* are the benefit expenditures incurred by the UI system from former employees of the firm, and *Payroll* is the firm's wage payments to employees. Thus, a firm's reserve ratio increases when it contributes relatively more or has fewer contributions to UI expenditures. That is, a higher reserve ratio implies a lower tax rate for the firm. Generally, the reserve ratio is computed as a 3-year moving average. Each state utilizing the Reserve ratio method then applies a specific formula for translating the reserve ratio (or changes in it) into a tax rate. These formulas also vary across states. The remaining methods (excluding Payroll decline) are all variants of this basic idea.

The specific calculation of the ERI is:

$$\text{ERI} = \left[\frac{\text{BEN} - (\text{IEC} + \text{IAC} + \text{NNC})}{\text{BEN}} \right] \times 100$$

BEN refers to total benefits charged in a given state. IEC represents “ineffective charges”. To compute these, employers are aggregated into 30 groups based on their experience factor. Within each group, the difference (excess) between benefits charged to the employers (*i.e.* benefits collected) and the benefits contributed by those employers. Summing over the 30 groups produces the IEC. It is a measure of how much of benefit expenditures are not completely financed by firm taxes. IAC represents benefits charged to employers who have gone out of business (and thus from whom no taxes may be collected). Finally, NNC represents benefits collected that were not charged to any particular employer.